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Report on the doctoral thesis

*Algebraic hyperstructures in the
model theory of valued fields*
by
Alessandro Linzi

The thesis of Mr. Linzi can be characterized by the following main points:

1. New results (Theorems 5.4 and 5.11) concerning relative quantifier elimination for henselian fields of residue characteristic 0 and of mixed characteristic (Theorems 5.30 and Corollary 5.32) are provided. These results extend the work of F.-V. Kuhlmann on amc-structures and embed it into the context of hyperfields. Moreover Mr. Linzi obtains original results on relative quantifier elimination for henselian fields of residue characteristic 0 in the language of graded rings (Theorems 5.18 and 5.19), apparently without forerunners in the literature.

2. It contains an almost encyclopedic and lucid study of the relationships between the notion of a hyperfield on one side and the following structures appearing in the model theory of valued fields on the other: leading term structures (J. Flenner), amc-structures (F.-V. Kuhlmann), angular component maps, and graded rings (induced by valuations).

3. A detailed investigation of the properties of valued hyperfields obtained by taking factor hyperfields of valued fields is presented, that supplements and elaborates recent results of J. Lee.

Summing up the doctoral thesis of Mr. Linzi demonstrates his broad knowledge of valued fields and hyperfields and their model theory, as well as his ability to develop mathematical concepts with the aim of extending a given mathematical framework. In my opinion the mathematical content of his well-written doctoral thesis exceeds considerably the requirements to justify the award of the degree of a Doctor of Philosophy in natural and exact sciences.

The submitted thesis fulfills the conditions set by Article 187 ust. 1-3 ustawy z dnia 20 lipca 2018 r. Prawo o szkolnictwie wyższym i nauce (Dz. U. z 2022 r. poz. 574 ze zm.).

Hagen Knaf