Summary of the Doctoral Thesis

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Title: Modified uncertainty relations from classical and quantum gravity
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A good hundred years after the necessity for a quantum theory of gravity was acknowledged by Albert Einstein, the search for it continues to be an ongoing endeavour. Nevertheless, the field still evolves rapidly as manifested by the recent rise of quantum gravity phenomenology supported by an enormous surge in experimental precision. In particular, the minimum length paradigm ingrained in the program of generalized uncertainty principles (GUPs) is steadily growing in importance.

The present thesis is aimed at establishing a link between modified uncertainty relations, derived from deformed canonical commutators, and curved spaces - specifically, GUPs and nontrivial momentum space as well as the related extended uncertainty principles (EUPs) and curved position space. In that vein, we derive a new kind of EUP relating the radius of geodesic balls, assumed to constrain the wave functions in the underlying Hilbert space, with the standard deviation of the momentum operator, suitably made compatible with the curved spatial background. This result is gradually generalized to relativistic particles in curved spacetime in accordance with the 3+1 decomposition, thereby relating semiclassical gravity with the EUP.

The corresponding corrections to the relation in flat space depend on the Ricci scalar of the effective spatial metric, the lapse function and the shift vector, as well as covariant derivatives thereof. The ensuing inequality is evaluated in Rindler, de Sitter and Schwarzschild backgrounds, at lowest approximation leading to identical effects, as well as to rotating geometries like Kerr black holes and their analogues in higher-order theories of gravity.

In a sense pursuing the inverse route, we find an explicit correspondence between theories yielding a GUP, possibly including a noncommutative geometry, and quantum dynamics set on non-Euclidean momentum space. Quantitatively, the coordinate noncommutativity translates to momentum space curvature in the dual description, allowing for an analogous transfer of constraints from the literature. However, a commutative geometry does not imply trivial dynamics; the corresponding types of GUP lead to a flat momentum space, described in terms of a nontrivial basis, permitting the import of further bounds.

Finally, we find a formulation of quantum mechanics which proves consistent on the arbitrarily curved cotangent bundle. Along these lines, we show that the harmonic oscillator can, given a suitable choice of operator ordering, not be used as a means to distinguish between curvature in position and momentum space, thereby providing an explicit instantiation of Born reciprocity in the context of curved spaces.

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