What Could a Pair of Universes Tell Us About the Multiverse?

Falsifying the Multiverse by our
Twin Antiuniverse Observational Effect

Author: **Samuel Barroso Bellido**

Supervisor: Prof. Mariusz P. Dąbrowski

—

Faculty of Pure and Exact Sciences
University of Szczecin

—

A Thesis
Submitted For The Degree of
Doctor of Physical Sciences

May 2022

Oświadczenie Doktoranta

Oświadczam, że moja praca pt.: **What could a pair of universes tell us about the multiverse?**

a. została napisana przeze mnie samodzielnie,
b. nie narusza praw autorskich w rozumieniu ustawy z dnia 14 lutego 1994 roku o prawie autorskim i prawach pokrewnych (Dz.U. 2019 r. poz. 1231) oraz dóbr osobistych chronionych prawem,
c. nie zawiera danych i informacji, które uzyskałem / uzyskałam w sposób niedozwolony,
d. nie była podstawą nadania tytułu naukowego lub zawodowego ani mnie ani innej osobie.

Ponadto oświadczam, że treść pracy przedstawionej przeze mnie do obrony, zawarta na przekazanym nośniku elektronicznym jest identyczna z jej wersją drukowaną.

Szczecin, dn. 17.05.2022 .... Samuel Barroso Bellido ....

podpis doktoranta

Oświadczenie

Wyrażam / nie wyrażam zgodę / zgody na udostępnienie mojej pracy doktorskiej pt.: **What could a pair of universes tell us about the multiverse?**

Szczecin, dn. 17.05.2022 .... Samuel Barroso Bellido ....

podpis doktoranta

Oświadczenie

Akceptuję ostateczną wersję pracy.

Szczecin, dn. 19.05.2022 .... Mariusz P. Dąbrowski ....

Author's Contribution

The research which concludes with the present piece of work gave rise to the articles:

[1] A. Balcerzak, S. Barroso Bellido, M. P. Dąbrowski and S. Robles Pérez. Phys. Rev. D **103**, 043507 (2021).

[2] S. Barroso Bellido. Phys. Rev. D **104**, 106009 (2021).

[3] S. Barroso Bellido, M. P. Dąbrowski. (arXiv: 2203.07069) (To be published)

[4] S. Barroso Bellido. The structure of the multiverse from the Entanglement Entropy. In *16th Marcel Grossmann Meeting on Recent Developments in Theoretical and Experimental General Relativity, Astrophysics, and Relativistic Field Theories.* (To be published)

[5] S. Barroso Bellido, F. Wagner. Phys. Rev. D **105**, 106001 (2022).

Part of the content in Chapters 2 and 3 is found in [1] and [2], Chapter 4 is written from the results in [3], and the discussions in Chapter 5 are taken from [4] and [5].

Abstract

The present work is aimed to falsify the multiverse as it is prescribed by the Third Quantization formalism of Canonical Quantum Gravity. The formalism naturally predicts pair creation of universes and so the possible existence of a twin antiuniverse of that we inhabit. We extensively study the quantum entanglement between them for different models, finding that it is relevant at the initial singularity, at the maxima and minima of expansion, and at some exotic singularities like the Little Rip singularity. We use the conclusions found from the entanglement research to constraint the interaction between our universe and its twin and thus recreate the semiclassical Friedmann equation from where we obtain the dynamics of the universe which takes the entanglement effects into account. We then apply it in order to get the observational imprints of our hypothetical twin antiuniverse on the spectrum of the cosmic microwave background. For the case we consider, the constant coupling which governs the strength of the interaction is calculated to be $\lambda_o \lesssim \mathcal{O}(10^{-56})\,\mathrm{s}^{-3}$ in order to reproduce the angular power spectrum obtained by Planck satellite. We finish by completing the Third Quantization formalism including a new particle in the multiverse accountable for the interaction between universes. That way, Third Quantization formalism is the true Quantum Field Theory it was supposed to be by construction.

Summary of the Doctoral Thesis

University of Szczecin – Institute of Physics – Samuel Barroso Bellido, M. Sc.

Title: What Could a Pair of Universes Tell Us About the Multiverse?

Supervisor: prof. dr. hab. Mariusz Dąbrowski

The present work is aimed to falsify the multiverse as it is prescribed by the Third Quantization formalism of Canonical Quantum Gravity. The formalism naturally predicts pair creation of universes and so the possible existence of a twin antiuniverse of that we inhabit. We extensively study the quantum entanglement between them for different models, finding that it is relevant at the initial singularity, at the maxima and minima of expansion, and at some exotic singularities like the Little Rip singularity. We use the conclusions found from the entanglement research to constraint the interaction between our universe and its twin and thus recreate the semiclassical Friedmann equation from where we obtain the dynamics of the universe which takes the entanglement effects into account. We then apply it in order to get the observational imprints of our hypothetical twin antiuniverse on the spectrum of the cosmic microwave background. For the case we consider, the constant coupling which governs the strength of the interaction is calculated to be $\lambda_o \lesssim \mathcal{O}(10^{-56})\,\mathrm{s}^{-3}$ in order to reproduce the angular power spectrum obtained by Planck satellite. We finish by completing the Third Quantization formalism including a new particle in the multiverse accountable for the interaction between universes. That way, Third Quantization formalism is the true Quantum Field Theory it was supposed to be by construction.

Keywords: Entanglement Entropy, Quantum Cosmology, Third Quantization.

Date, signature:

Streszczenie rozprawy doktorskiej

Uniwersytet Szczeciński – Institut Fizyki – mgr. Samuel Barroso Bellido

Tytuł: What Could a Pair of Universes Tell Us About the Multiverse?

Promotor: prof. dr. hab. Mariusz Dąbrowski

Celem rozprawy doktorskiej jest falsyfikacja koncepcji multiwszechświata uzyskanej w ramach procedury trzeciego kwantowania w formaliźmie kanonicznej kwantowej grawitacji. Ta procedura w sposób naturalny przewiduje kreację pary wszechświatów a zatem w szczególności możliwe istnienie bliźniaczego antywszechświata do tego, w którym my zamieszkujemy. Poprzez intensywne badanie kwantowego splątania pomiędzy naszym oraz bliźniaczym wszechświatem dla różnych modeli, wyciągamy wniosek, iż splątanie odgrywa istotną rolę w takich punktach ewolucji jak: osobliwość początkowa, maksima i minima ekspansji, a także niektóre egzotyczne osobliwości jak m.in. Małe Rozerwanie. Otrzymane rezultaty pozwalają na wymodelowanie oddziaływania pomiędzy parą wszechświatów i skonstruowanie półklasycznego równania Friedmanna pozwalającego na określenie dynamiki naszego wszechświata z uwzględnieniem kwantowego splątania. To pozwala na obliczenie wpływu istnienia hipotetycznego bliźniaczego wszechświata na spektrum obserwowanego kosmicznego mikrofalowego promieniowania tła. Porównanie z danymi z satelity Planck daje ograniczenie na stałą sprzężenia definującą oddziaływanie pary wszechświatów jako $\lambda_o \lesssim \mathcal{O}(10^{-56})\,\mathrm{s}^{-3}$. Na zakończenie wprowadzamy udoskonalenie procedury trzeciego kwantowania poprzez wprowadzenie nowej cząstki pośredniczącej dla oddziaływania wszechświatów w multiwszechświecie. W ten sposób trzecie kwantowanie nabiera zupełnych cech kwantowej teorii pola, co jest zgodne z zasadniczą intencją tej rozprawy.

Słowa Kluczowe: Entropia Śplątania, Kosmologia Kwantowa, Trzecie Kwantowanie.

Data, podpis:

Acknowledgements
Agradecimientos

Los agradecimientos están escritos en español para todos aquellos que lean español, y en inglés para todos los que no. Si he olvidado a alguien, no se sientan ofendidos.

The acknowledgements are written in Spanish for those who can read Spanish, and in English for those who cannot. If I forgot someone, do not feel offended.

Primero, y antes que nada, los agradecimientos son para mi padre por casi 30 años de existencia. Si queríais estar antes que él, pues habed estudiado. No hay mucho que explicaros a ninguno que no sepáis ya.

Segundo a Ana, a mi Ana. Primero, por hacer las dibujos de la tesis, y luego... pues aquí sí hay mucho que explicaros pero tampoco es el lugar. Unos por darte la vida y otros por cambiártela radicalmente. Horas de comprensión, risas y llantos terminan en una relación que, pase lo que pase, va a terminar durando para siempre, ya sea romántica o de pura amistad. Nadie que haya tragado tanta mierda y haya actuado en pos de mi mejora personal se merece el más mínimo desprecio. Tienes mi más sincera gratitud y amistad de por vida. ♡

Ahora es el turno de mis amigos de toda la vida. Ahora toca la gente que, después de Ana, nadie entiende por qué están a mi lado durante tanto tiempo. Tampoco yo. Entiendo que a mi padre le ha caído una gorda ya que creo que es ilegal dejar a tu hijo en la cuneta, pero... ¿ustedes? Supongo que el roce hace que una esfera funcione bien como pieza en un castillo de Lego... y son muchos años rozando. Es más fácil creerse la existencia de otro universo que esto, pero cada uno se jode la vida como quiere. Ante mi más profundo asombro... en orden analfabético para que nadie se queje del orden: $\frac{1}{\sqrt{3}}|\text{Sergio} + \text{Ale} + \text{Jesi}\rangle$, Esther, Isa, Irene, Aurelia, Jose... os doy las gracias a todos.

También, a Trini que durante estos últimos años ha estado ahí con nosotros. Y otros, a quien considero importantes, que han estado ahí en menor medida, pero que he aprendido cosas de ellos o con los que me he divertido durante todo este tiempo, como Antoñito, Ale Peliazul, Jechu, Rasher, Fernando, Ernesto, Juan, Loló, y algunos otros que no son ni de este continente como Jorge.

Y antes de cambiar al inglés, agradezco a Salva y a Mar las conversaciones amenas, otras no tan amenas, emails, cartas de recomendación y los momentos en el bar, unos con cervezas y otros con cafés. Espero que siga siendo así tanto tiempo como se pueda.

Finally we turn into English. For their almost daily support, I thank my teammates for trying to take me out of the physical and mental cavern I created at home and downstairs. In an attempt to create the most advance society of Physics in the world we created the renowned Doubly Informal Seminar, where the ratio beers over Physics content... well, it is better not to talk about it. Such was the case we slowly disbanded it as a Seminar and it was moved to a coffee conversation around 5 pm.

Among my teammates, undoubtedly, Fabian has been the cornerstone during these four years. I have learnt so many things around him, breathing the same air, and getting hit constantly by any of his ideas. I appreciate every single moment he put me on that spot of anxiety craving for my answer with his eyes. That is the best way to learn. Also, I am really grateful for his help with the incomprehensible tantrums of Mathematica, his thoughts on my work, and our late collaboration. Besides, Enrico, who shares most of my lifestyle but in an Italian way, Paolo, Sara, Roberto, and those who were there and I saw them a few times or during a short period: Alessandro and Filippo.

I would like to express my absolute gratitude to the whole Cosmology Group in

Szczecin. Mainly, and the reason why I am here writing the PhD thesis, to my supervisor Mariusz, who let us all to carry out our studies there with all the opportunities it entails. Also, Enzo and Adam who helped us with the awful administrative part of the University and some good times I have in memory.

And lastly, I finished writing this text in Bologna, where I have been cordially hosted by Roberto and Alexandr to whom I am very grateful as well as I am to the whole department.

Conventions and List of Symbols

- Regarding the writing style, I preferred to take the whole responsibility of what it is written. That is why I use the pronoun *'I'* when something was done by myself to differentiate it from the case when something was made in collaboration with my teammates or it is customary in the literature, for which I employ the pronoun *'we'*. This preference can be understood as impolite for which I apologize right now.

- As usual, greek indices $(\mu, \nu, ...)$ are reserved for spacetime components and latin indices $(i, j, ...)$ for spatial components.

- Equations are written in natural units, $c = \hbar = 1$, except where it is indicated.

- Einstein summation is assumed when indices are repeated in a proper covariant way.

- In general, unless specified, we used the sign criterion $(-, +, +, +)$.

- Curvature tensors and derived ones are defined as follows.

  - Levi-Civita connection:

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\lambda} \left(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu} \right).$$

  - Riemann tensor:

$$\mathcal{R}_{\mu\nu\rho}{}^{\lambda} = \partial_\mu \Gamma^{\lambda}_{\nu\rho} - \partial_\nu \Gamma^{\lambda}_{\mu\rho} + \Gamma^{\lambda}_{\mu\sigma} \Gamma^{\sigma}_{\nu\rho} - \Gamma^{\lambda}_{\nu\sigma} \Gamma^{\sigma}_{\mu\rho}.$$

  - Ricci tensor:

$$\mathcal{R}_{\mu\nu} = \mathcal{R}_{\mu\lambda\nu}{}^{\lambda}.$$

- Ricci scalar:

$$\mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}.$$

- Einstein tensor:

$$\mathcal{G}_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R}.$$

- Einstein field equations (in the most fundamental manner):

$$\mathcal{G}_{\mu\nu} = \kappa\mathcal{T}_{\mu\nu}.$$

- Dirac notation and function notation will be employed indifferently.

Physical Variables:

L	Lagrangian
$\mathcal{L}$	Lagrangian density
$\mathcal{H}$	Hamiltonian
H	Hubble parameter
$\mathfrak{H}$	Hilbert spaces, except in Section 4

Mathematical Symbols:

$\propto$	Proportional to
x^*	Complex Conjugated of x.
$x^\dagger$	Conjugate Transpose of x.
x^{T}	Transpose of x.
$\delta(x - x_o)$	Dirac Delta at $x = x_o$.
$\dim(X)$	Dimension of X.
$\emptyset$	Empty Set.
$\mathbb{C}$	Set of Complex Numbers.
$\mathbb{N}$	Set of Natural Numbers.
$\mathbb{R}$	Set of Real Numbers.
$\mathbb{R}^+$	Set of Positive Real Numbers.
$\mathbb{X}^*$	Set $\mathbb{X} \cup \{0\}$
$\{x_i\}$	Set of Elements x_i
$L^2(\mathbb{R})$	Lebesgue Space L^2
∂_μ	Partial Derivative.
∂_μ^2	Second Partial Derivative.
∇_μ	Covariant Derivative.
∇^2	Laplacian Operator.
$\Box = \partial_\mu\partial^\mu$	D'Alambert Operator.
f^{-1}	Inverse of f.
i	Imaginary Unit.
e	Euler's Constant: 2.71828...

Contents

1 Introduction

Before I was a student enrolled in Physics, I perfectly knew that there was no need for seeing something to guess an explanation of its behavior. I was aware that negative electrons moved around a positive atomic nucleus, whose behavior was ruled by something called Quantum Mechanics, even though I had not heard what the Schrödinger equation was. After my courses on Quantum Mechanics and Quantum Field Theory, I realized that I was correct. There was something called electron, a fundamental particle, whose particular motion and fundamental interactions with photons were derived from the Lagrangian of a $U(1)$ gauge theory named QED (see e.g. [1, 2]). Supposedly, we have never seen an electron and we will never see it since it is a point particle. In an mathematical way of thinking, one can say that something which is zero-dimensional, as this electron, could be *physical* because it can built reality as in analogy with the real line from isolated points. However, we argue that what it is important is just its effects, like its interaction at a distance with other particles. The actual presence of those precise effects *defines* the notion of electron, even if we have never isolated a single one due to vacuum polarization or whatever other physical reason. Even stronger arguments than these can be given for virtual particles. That way of proceeding is what I will use to obtain potential evidences of the existence of other universes.

Here, I present a dissertation grounded on the theory of Canonical Quantum Gravity, originally developed by DeWitt[3–5] from previous ideas by Dirac [6] and Bergmann [7] about the Hamiltonian formalism for gravitation, the ADM formalism by Arnowitt, Deser and Misner [8], and Wheeler's novel theory of geometrodynamics [9]. With this as our fundamental theory, we build a level III kind of multiverse, adopting the classification by Tegmark [10], using the so-called Third Quantization of Canonical Quantum Gravity [11]. Our multiverse is going to be an entity where universes are excited states of a field of universes with different properties. Indeed, Third Quantization is nothing more than an analogy to Quantum Field Theory but using universes as particles. We will only consider the creation of a pair of universes from *nothing*, it is, a certain quantum vacuum defined for the field of universes, where space and time are supposed to be non-existent. Pair creation is expected to be the most natural way to create universes, since the three-leg vertex is the most probable interaction in a field theory. As it happens to the spin of an electron and its antiparticle, the positron, after the decay of some neutral vector boson, their spins are entangled, and hence both universes by analogy. The analysis of the entanglement of the pair is our aim, as it is its effects on the internal dynamics of each universe, i. e., how it changes the Friedmann equations. In this manner, we could test the existence of a twin partner of our universe just looking at its effects over our very universe, as in the case of the electron. The deviation from the dynamics ruled by the standard ΛCDM model implies a variation of the cosmological parameters, the primordial fluctuations, and therefore the power spectrum of the photons from the Cosmic Microwave Background (CMB). Proceeding as it is usually done, we can falsify our model by comparing the expected power spectrum and the observed by Planck satellite [12], and constraint the parameters of the theory for which the model fits the observational data.

There have been many other studies which include the interaction between infinite or a very high amount of universes [13–20]. All of them, except in [20], do not treat a level III multiverse but a level II multiverse, in which separated patches of the spacetime of the same universe are causally disconnected. However, they also find that the dynamics of each patch is affected by the existence of the other disconnected regions as we find it for level III multiverses. In comparison to those other studies, we consider only the most probable picture in the Third Quantization scenario, which is the pair creation of universes, where the interaction is just between two universes, that is, the most simple one.

The structure of the present work is as it follows. I start reviewing the fundamental points we are going to need on Canonical Quantum Gravity and the Third Quantization picture in Chapter 2. In the same chapter, I explain how we find the entanglement entropy of the pair of universes, which gives a qualitative measure of the state of entanglement in which the bipartite system is. In Chapter 3, I analyze some general properties of the entanglement entropy and its behavior for different cosmological models during the evolution of the universe, i. e., as the time goes by in a cosmological and classical sense. The observational imprints and analysis on the CMB due to the interactions with our hypothetical twin universe is found in Chapter 4. In Chapter 5, a critical analysis of my own results is given and a correction to the Third Quantization formalism is proposed to improve its paradigm. Finally, in Chapter 6, I conclude with a summary of everything we found, and give some prospectives of my work.

2

Third Quantization and the Pair Creation of Universes

2.1 Introduction

As in any text in Physics, I start examining the theoretical foundations to what it comes later on. Along the exposition of my results I will need to use some fundamental concepts of General Relativity, Quantum Mechanics and Quantum Field Theory. Thus, this chapter is going to be dedicated to remind all the necessary concepts and results of each theory we will employ and to which I will be referring in future chapters.

2.1.1 General Relativity and Cosmology

The theory of General Relativity [21–23] is a well established theory of gravitation which has been able to explain with high precision what we see at very large scales, for instance, the recent observations of merging black holes and gravitational waves.

Its application to Cosmology has also achieved many important goals, like the description of the anisotropies of the CMB. The implication of an inflationary era to explain those anisotropies requires the existence of some energy which exerts a repulsive gravitational effect violating the strong energy condition [24]. The origin of such energy is sometimes attributed to a scalar field, known as the inflaton, whose potential energy is expected to be almost constant for a long period of that inflationary era. The observational data also permits the existence of a cosmological constant Λ whose origin is not yet known. Discussions on the origin of the cosmological constant will be avoided here. It must be included as a new term in the original field equations of gravity to built the complete Einstein field equations

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} + \Lambda g_{\mu\nu} = \kappa \mathcal{T}_{\mu\nu}, \tag{2.1}$$

where $\kappa = 8\pi G$, and G is the gravitational constant.

The action from which we can derive this equation of motion is

$$S_{\mathrm{EH}} = S_g + S_m = \int \mathrm{d}^4x\sqrt{|g|}\left[\frac{1}{2\kappa}(R - 2\Lambda)\right] + \int \mathrm{d}^4x\sqrt{|g|}\mathcal{L}_m, \tag{2.2}$$

where $g = \det(g_{\mu\nu})$, and $\mathcal{L}_m$ is the Lagrangian density of the matter fields, whence we define the stress-energy tensor

$$\mathcal{T}_{\mu\nu} = -\frac{2}{\sqrt{|g|}}\frac{\delta\left(\sqrt{|g|}\mathcal{L}_m\right)}{\delta g^{\mu\nu}}. \tag{2.3}$$

Eq. (2.2) is known as the Einstein-Hilbert action where we have included the matter fields and the cosmological constant.

The derivation of the Einstein field equations (2.1) from the action (2.2) needs attention on the boundary. Applying the principle of least action to find the equation of motion for $g^{\mu\nu}$ into the manifold $\mathcal{M}$, we find

$$\frac{\delta S_{EH}}{\delta g^{\mu\nu}} = \frac{1}{2\kappa}\left[\int_{\mathcal{M}} \mathrm{d}^4x\sqrt{|g|}\left(\mathcal{G}_{\mu\nu} + \Lambda g_{\mu\nu}\right) + \int_{\mathcal{M}} \mathrm{d}^4x\sqrt{|g|}g^{\alpha\beta}\frac{\delta R_{\alpha\beta}}{\delta g^{\mu\nu}}\right] + \int_{\mathcal{M}} \mathrm{d}^4x\frac{\delta\left(\sqrt{|g|\mathcal{L}_m}\right)}{\delta g^{\mu\nu}}, \tag{2.4}$$

from where we recover Eq. (2.1) if and only if the second term in brackets disappears. That term is generally vanishing when we consider a static metric on the boundary, otherwise we need to include a correction term to cancel the value of that integral. This condition is necessary if we work with compact manifolds or we do not want to constraint the properties of the metric anywhere. Working that term out a bit, it is

$$\frac{1}{2\kappa}\int_{\mathcal{M}} \mathrm{d}^4x\sqrt{|g|}g^{\alpha\beta}\frac{\delta R_{\alpha\beta}}{\delta g^{\mu\nu}} = \frac{1}{\kappa}\int_{\partial\mathcal{M}} d^3x\epsilon\sqrt{|h|}K, \tag{2.5}$$

where $\partial\mathcal{M}$ is the boundary of the manifold $\mathcal{M}$, the variable $K = \nabla_\mu n^\mu$ is the extrinsic curvature, n^μ is a normal vector to $\partial\mathcal{M}$, and $\epsilon = n_\mu n^\mu = \pm 1$. It is called the Gibbons-Hawking-York (GHY) term, and gives a generalization of the Einstein-Hilbert action (2.2) as

$$S_{\text{EH}} = \int \mathrm{d}^4x\sqrt{|g|}\left[\frac{1}{2\kappa}(R-2\Lambda)\right] - \frac{1}{\kappa}\int_{\partial\mathcal{M}} d^3x\epsilon\sqrt{|h|}K + \int \mathrm{d}^4x\sqrt{|g|}\mathcal{L}_m. \tag{2.6}$$

This generalized action will be used not to be mistaken due to boundary effects.

In regard to Cosmology [25], the ordinary way to study our universe is by virtue of a homogeneous and isotropic portrayal given by the four-dimensional Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \frac{1}{1-\mathtt{k}r^2}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2\right], \tag{2.7}$$

where $a(\eta)$ is the scale factor, used to be accountable for the relative size of the universe, $\mathrm{d}\Omega_2^2$ is a differential section of the two-dimensional solid angle in spherical coordinates, the index of curvature and the conformal time are defined, respectively, like

$$\mathtt{k} = \begin{cases} 1, & \text{for a closed universe} \\ 0, & \text{for a flat universe} \\ -1, & \text{for a open universe} \end{cases}, \qquad \text{and} \qquad \mathrm{d}\eta = \frac{\mathrm{d}t}{a}, \tag{2.8}$$

and t is the cosmological time. To maintain the essence of the model, a perfect fluid is introduced as the matter content described by a stress-energy tensor like

$$T^{\mu\nu} = \mathrm{diag}(\rho, p, p, p), \tag{2.9}$$

where ρ is the energy density and p is the pressure of the fluid.

The equations of motion obtained from the field equations (2.1) with the metric (2.7) and the stress-energy tensor (2.9) are the Friedmann equation and the Raychaudhuri equation

$$H^2 = \frac{\kappa}{3}\rho - \frac{\mathtt{k}}{a^2} + \frac{\Lambda}{3}, \qquad \text{and} \qquad \frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho+3p) + \frac{\Lambda}{3}, \tag{2.10}$$

respectively. With these two equations, the dynamics of the universe can be found once the total content is fixed.

The equation of state of the perfect fluid in the simplest barotropic form is

$$p = \omega\rho, \tag{2.11}$$

where ω is the barotropic parameter. A simple analysis of the dynamics ruled by Eqs. (2.10) with this equation of state comes to the conclusion that there must be a singularity at the very beginning of the expansion of the universe [26], popularly called Big Bang singularity. At this singularity, the pressure and the density of the universe is expected to diverge classically. However, for different equations of state, that can be also of the barotropic kind $p(\rho)$, or giving non-standard values to ω in (2.11), it is, out of the interval $\omega \in [-1, 1]$, one can find that the universe shows some exotic singularities (a compilation of some of them can be found in [27]). Those exotic singularities are classified by their different properties. At the specific time in which they appear, the universe can be in any state of pressure and density, for instance, moments when the pressure and the density of the fluid vanish in the universe.

It is standard in Cosmology to recognize perfect fluids as a scalar field $\phi(t)$ whose pressure and density are

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \qquad \rho(\phi) = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{2.12}$$

where $V(\phi)$ is the potential energy of the field.

2.1.2 Quantum Theory

The description of the large scales through the theory of General Relativity keeps as fundamental the determinism Physics had always enjoyed. Despite the success of Physics as a deterministic science when applied for the classical world, its description of the microscopic world was found to be fully probabilistic. The lost of the usual kind of determinism and the new probabilistic operation of the more essential components of reality has as implication the necessity of a philosophical interpretation.

There were many physicist going after the theory, but it is said that its parents are Heisenberg [28] and Schrödinger [29], who describe equivalent ways of writing the theory. Quantum Mechanics is the microscopic theory which defines an essential object called the wave function of particles or systems of them. It is a mathematical entity, living in $L^2(\mathbb{R})$ except for some physically unrealistic ones, which is built upon quantum states $|\phi_i\rangle$ and their probabilities p_i like

$$|\Psi(\vec{x},t)\rangle = \sum_i \sqrt{p_i(t)}\,|\phi_i(\vec{x},t)\rangle\,. \tag{2.13}$$

Schrödinger found it to fulfill a wave equation equivalent to the conservation of the energy $E = p^2/2m + V$, which Galileo stated in words [30, 31]. It is called the Schrödinger equation

$$\left[-\frac{1}{2m}\nabla^2 + V(\vec{r},t)\right]\Psi(\vec{r},t) = \mathrm{i}\frac{\partial}{\partial t}\Psi(\vec{r},t), \tag{2.14}$$

where m is the mass of the particle for which the wave function is $\Psi(\vec{r},t)$ in the position representation. Here the quantization à la Dirac [32] of the position, linear momentum and the energy

$$\hat{x} = x, \qquad \hat{p}_i = -\mathrm{i}\frac{\partial}{\partial x^i}, \qquad \hat{H} = \mathrm{i}\frac{\partial}{\partial t}, \tag{2.15}$$

respectively, has been applied.

Schrödinger equation (2.14) comes from the action

$$S_{\mathrm{Sch}} = \int \mathrm{d}^4x \left[\Psi^\dagger\left(\mathrm{i}\frac{\partial}{\partial t} + \frac{1}{2m}\nabla^2\right)\Psi - V\Psi^\dagger\Psi\right], \tag{2.16}$$

when we apply the variational principle with respect to $\Psi^\dagger$.

It is interesting to point out that Quantum Mechanics is written on flat space and without any relativistic consideration. It is, Minkowski spacetime, described by the metric

$$\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1), \tag{2.17}$$

is not even considered.

On the quest for a relativistic Quantum Mechanics, Klein [33] and Gordon [34] found out, accidentally, an equation of motion for complex scalar fields

$$\left[\Box - m^2\right]\Psi(\vec{r},t) = 0, \tag{2.18}$$

where $\Psi(\vec{r},t)$ is the complex scalar field, which is not related to any probability amplitude. This is a quantum version of the relativistic equation for the conservation of the energy $E^2 = m^2 + p^2$. After a Fourier transform

$$\Psi(\vec{r},t) = \int \mathrm{d}^3k \Psi(\vec{k},t) \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}}, \tag{2.19}$$

where $\Psi(\vec{k},t)$ is the wave function in the momentum representation, Eq. (2.18) is converted into

$$\left[\partial_t^2 + k^2 - m^2\right] \Psi(\vec{k},t) = 0. \tag{2.20}$$

It admits two conjugated solutions $\Psi_k^{(\dagger)} \propto \mathrm{e}^{\pm\mathrm{i}\omega t}$, where the dispersion relation $\omega^2 = k^2 + m^2$ holds. The Klein-Gordon (KG) equation (2.18) is therefore the equation of motion of the field $\Psi^\dagger$ whose action is

$$S_{\mathrm{KG}} = \int \mathrm{d}^4x \left(-\eta^{\mu\nu}\partial_\mu\Psi^\dagger\partial_\nu\Psi - m^2\Psi^\dagger\Psi\right), \tag{2.21}$$

which is invariant under global $U(1)$ transformations. The conserved Noether current associated with the $U(1)$ symmetry here shows that both solutions have opposite conserved electric charges, representing a pair *particle* and *antiparticle*. Choosing the standard convention, we will note particles as Ψ and antiparticles as $\Psi^\dagger$ from now on.

2.1.3 Quantum Field Theory

The field described by the KG equation (2.18) is still a classical field depending on the coordinates $(\vec{r},t)$. The so-called second quantization of Quantum Mechanics avoids the usage of spacetime coordinates to simplify the analysis of interacting fields. That is the matter of a Quantum Field Theory.

In order to construct it, same way a particle is described by a vector $|\Psi\rangle$ into a certain Hilbert space [35], a system of many particles is described by a Fock state into a Fock space. A Fock state represents the system of a variable number of particles, each one with its own properties. In general, one could denote it as $|n_{k_1}, n_{k_2}, \cdots\rangle$, where n_{k_i} is the number of particles in the state k_i. It makes it the most natural way to think of a basis for a Fock space.

The annihilation and creation operators, $\hat{a}$ and $\hat{a}^\dagger$, respectively, are physically essential here. They act as expected

$$\hat{a}_k |n_k\rangle = \sqrt{n_k}\, |n_k - 1\rangle , \qquad \hat{a}_k^\dagger |n_k\rangle = \sqrt{n_k + 1}\, |n_k + 1\rangle , \tag{2.22}$$

annihilating a particle in the state k or creating a new one. The Fock space is then built once the vacuum state $|0\rangle$ is defined such that it is annihilated by the annihilation operator: $\hat{a}\,|0\rangle = 0$. The right ordered operation of both defines the number operator $\hat{N}_k$ acting like

$$\hat{N}_k |n_k\rangle = \hat{a}_k^\dagger \hat{a}_k |n_k\rangle = n_k |n_k\rangle , \tag{2.23}$$

whose eigenvalue is the number of particles n_k in the state k.

Thus, a complex quantum scalar field is a functional written in terms of the annihilation and creation operators for the positive ($\hat{a}$ and $\hat{a}^\dagger$) and negative ($\hat{b}$ and $\hat{b}^\dagger$) modes

$$\hat{\Psi}(\vec{r}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3 \sqrt{2E_k}} \left(\hat{a}_k \mathrm{e}^{-\mathrm{i}\vec{k}\vec{r}} + \hat{b}_k^\dagger \mathrm{e}^{\mathrm{i}\vec{k}\vec{r}} \right). \tag{2.24}$$

The state $\hat{a}^\dagger |0\rangle$ is the 1-particle state, and $\hat{b}^\dagger |0\rangle$ is another 1-particle state. Choosing the convention in which $\hat{a}^\dagger |0\rangle$ is the 1- particle state, then $\hat{b}^\dagger |0\rangle$ represents its antiparticle whose charges are opposite.

It will be useful to know the relation between two different bases, let us say $\{|u_i\rangle\}$ and $\{|\bar{u}_j\rangle\}$, in which one can describe a quantum state $|\Psi\rangle$:

$$|\Psi\rangle = \sum_i \left[\hat{b}_i |u_i\rangle + \hat{b}^\dagger |u_i\rangle^* \right] = \sum_j \left[\hat{c}_j |\bar{u}_j\rangle + c^\dagger |\bar{u}_j\rangle^* \right], \tag{2.25}$$

where $\hat{b}$ and $\hat{c}$ are the annihilation operators for both bases. A relation between both bases and its inverse, called the Bogoliubov transformations [36], exist as

$$|\bar{u}_j\rangle = \sum_i \left[\alpha_{ji} |u_i\rangle + \beta_{ji} |u_i^*\rangle \right], \qquad |u_j\rangle = \sum_j \left[\alpha_{ji}^* |\bar{u}_j\rangle - \beta_{ji} \left|\bar{u}_j^*\right\rangle \right], \tag{2.26}$$

where the matrices α_{ij} and β_{ij} are known as the Bogoliubov coefficients, fulfilling

$$\sum_k \left(|\alpha_{ij}\alpha_{ik}^*| - \left|\beta_{ij}^* \beta_{ik}\right| \right) = \delta_{ij}. \tag{2.27}$$

Also, a relation for the ladder operators can be given as

$$\hat{b}_i = \sum_j \left[\alpha_{ji} \hat{c}_j + \beta_{ji}^* \hat{c}_j^\dagger \right], \qquad \hat{c}_i = \sum_i \left[\alpha_{ji}^* \hat{b}_i - \beta_{ji}^* \hat{b}_i^\dagger \right], \tag{2.28}$$

where the relations

$$\langle u_i|u_j\rangle = \delta_{ij}, \qquad \left\langle u_i^*\middle|u_j^*\right\rangle = -\delta_{ij}, \qquad \left\langle u_i\middle|u_j^*\right\rangle = 0, \tag{2.29}$$

have been expected for the right inner product.

It is clear that the definition of a certain vacuum $|0\rangle$ is unique for a certain normalized basis $\{|u_i\rangle\}$, and it is not the vacuum for a different one $\{|\bar{u}_j\rangle\}$. It is why we find that

$$\left\langle\bar{0}\middle|N_i\middle|\bar{0}\right\rangle = \sum_j \left|\beta_{ji}^2\right|, \tag{2.30}$$

which is the number of particles in the u_i-modes present in the vacuum state $\left|\bar{0}\right\rangle$.

And last but not least, we would like to remember that the interactions in Quantum Field Theory are studied at the perturbative level. It implies that any coupling constant which controls the strength of the interaction must be very small. For instance, let us consider the Lagrangian density for interacting real scalar field up to the fourth order

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\Psi\partial^\mu\Psi - \frac{1}{2}m^2\Psi^2 - \frac{\lambda_3}{3!}\Psi^3 - \frac{\lambda_4}{4!}\Psi^4. \tag{2.31}$$

The coupling constants $\lambda_{i=\{3,4\}}$, must be much smaller than one for the amplitude probabilities to converge after summation of all possible processes including loops. As it happens for Quantum Chromodynamics, it is possible for the coupling constants to be close to the unity or bigger, what would make us to use non-perturbative methods. Luckily, it will not be our case at any point.

2.2 Canonical Quantum Gravity

2.2.1 General Theory and Discussion

In our research, the theory we employ is the Canonical Quantum Gravity (CQG) theory, conceived by DeWitt [3–5] in 1967. It is well known to be one of the first quantum gravity theories which yields some essential results when it is applied to Cosmology,

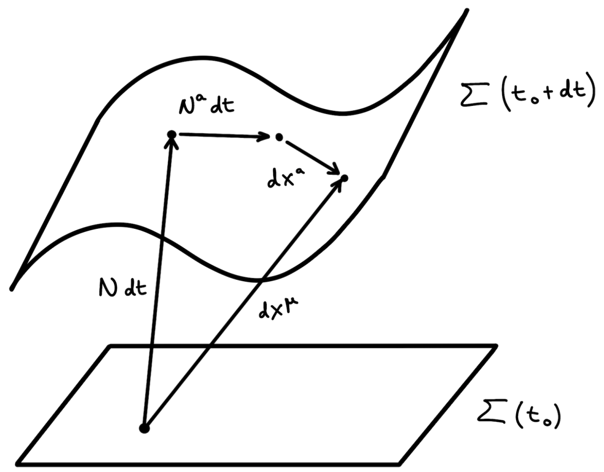

Figure 2.1: ADM foliation of the spacetime.

setting off the field known as Quantum Cosmology (QC). This branch of Physics has been extensively investigated during the 80's, initiated by the seminal works by Vilenkin [37] in 1982 and Hartle and Hawking [38] in 1983.

In QC, one of the most important results is the existence of a universal wave function which takes into account all the infinite degrees of freedom of the universe. The paradigm of the theory is kept in line with the Many-Worlds Interpretation by Everett [39], for which DeWitt expressed his liking, and his theory, to be in agreement with that new interpretation of Quantum Mechanics [40]. A good discussion about it can be found in [41].

The mathematical formulation of the theory starts by applying the ADM foliation of the spacetime. A spacetime described by a general metric $g_{\mu\nu}$ will be decomposed as [42, 43]

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = \left(N_a N^a - N^2\right)\mathrm{d}t^2 + 2N_a\mathrm{d}t\mathrm{d}x^a + h_{ab}\mathrm{d}x^a\mathrm{d}x^b, \tag{2.32}$$

where h_{ab} is the spatial part of the metric $g_{\mu\nu}$, called the 3-metric, $N(t)$ is called the lapse function and $N^a(t)$ is the shift vector. The simple geometrical meaning of those vectors can be understood taking a look at Figure 2.1. The differential distance between two events was considered to be

$$\mathrm{d}x^\mu = (N\mathrm{d}t, N^a\mathrm{d}t + \mathrm{d}x^a)^{\mathrm{T}}. \tag{2.33}$$

Hence, the generalized coordinates are now $(h_{ab}, N, N^a, \{\varphi\})$, where $\{\varphi\}$ are the matter degrees of freedom.

The gravitational part of the Einstein-Hilbert action (2.6) is written with the ADM metric (2.32) like

$$S = \frac{1}{2\kappa}\int \mathrm{d}^4x \left[G^{abcd}K_{ab}K_{cd} + \sqrt{h}\left({}^{(3)}R - 2\Lambda\right)\right], \tag{2.34}$$

where G_{abcd} is the DeWitt's metric

$$G^{abcd} = \frac{\sqrt{h}}{2}\left(h^{ac}h^{bd} + h^{ad}h^{bc} - 2h^{ab}h^{cd}\right), \tag{2.35}$$

$K_{ab} = \nabla_a n_b$ is the extrinsic curvature, n^μ is the normal vector to the hypersurface $\Sigma(t)$ satisfying $n_\mu n^\mu = -1$, ${}^{(3)}R$ is the Ricci scalar derived from the 3-metric h_{ab}, and h is the determinant of h_{ab}. From (2.34) we check that the coordinates N and N^a are non-dynamical, but constraints, since

$$p_N = \frac{\partial\mathcal{L}}{\partial\dot{N}} = 0, \qquad p_{N^a} = \frac{\partial\mathcal{L}}{\partial\dot{N}^a} = 0. \tag{2.36}$$

The associated momentum to the spatial metric, which is the only gravity-related dynamical variable, is

$$p^{ab} = \frac{\partial\mathcal{L}}{\partial\dot{h}_{ab}} = \frac{1}{16\pi}\left(K^{ab} - Kh^{ab}\right), \tag{2.37}$$

where $K = K_{ab}h^{ab}$ is the trace of K_{ab}. The action (2.34), after a straightforward calculation, is

$$S = \frac{1}{2\kappa}\int \mathrm{d}^4x \left(p^{ab}\dot{h}_{ab} - N\mathcal{H} - N^a\mathcal{H}_a\right), \tag{2.38}$$

from where we identify N and N^a to play the part of Lagrangian multipliers and thus Eqs. (2.36) give the relations

$$\mathcal{H} \propto G_{abcd}p^{ab}p^{cd} - \frac{\sqrt{h}}{2\kappa}\left({}^{3}R - 2\Lambda\right) = 0, \qquad \mathcal{H}_a \propto \nabla_b p_a{}^b = 0. \tag{2.39}$$

The first equation is the constraint of the infinite-dimensional phase space to a hypersurface whose Hamiltonian vanishes to which one should include the matter terms. Each point in such phase space corresponds to a universe with a certain 3-metric, a certain matter content, and some state of motion, generally speaking. The second equations is not as important as the first one as it only imposes a condition on the momenta p^{ab}. Keeping in mind the idea of quantizing gravity through the most natural procedure, promoting the canonical coordinates and their momenta to operators is the aim of CQG. The quantization is made à la Dirac with the operators

$$\hat{h}_{ab} = h_{ab}, \qquad \hat{p}^{ab} = -\mathrm{i}\frac{\partial}{\partial h_{ab}}, \qquad \{\hat{\varphi}\} = \{\varphi\}, \qquad \{\hat{p}_{\varphi}\} = \left\{-\mathrm{i}\frac{\partial}{\partial \varphi}\right\}. \tag{2.40}$$

The resulting equation is the famous *Wheeler-DeWitt* (WDW) equation [3, 42]

$$\mathcal{H}\Psi = \left\{-2\kappa G_{abcd}\frac{\partial^2}{\partial h_{ab}\partial h_{cd}} + \frac{\sqrt{h}}{2\kappa}\left[-^{(3)}R + 2\Lambda + 2\kappa\hat{\mathcal{T}}^{00}\right]\right\}\Psi(h_{ij}, \{\varphi\}) = 0, \tag{2.41}$$

where we have included the matter contribution and a new function $\Psi(h_{ij}, \{\varphi\})$ called the wave function of the universe. If the matter content is just a combination of non-interacting scalar fields, we recognize

$$\hat{\mathcal{T}}^{00} = \sum_n \left[-\frac{1}{2h}\frac{\partial^2}{\partial \varphi_n^2} + \frac{1}{2}h^{ij}\partial_i\varphi_n\partial_j\varphi_n + V(\varphi_n)\right]. \tag{2.42}$$

About Eq. (2.41), there are many things to point out, and I will expound shortly some of them:

- A very important mathematical detail arises when the kinetic terms are quantized. It is called the *factor ordering problem*[44–46]. It is about the choice among all possible permutations one could make to quantize $p^{ab}p^{cd}$ in the first of Eqs. (2.39). Classically, p^2 is equivalent to any other term such as $xp^2\frac{1}{x}$, or $g^{\mu\nu}p_\mu p_\nu$ to $p_\mu g^{\mu\nu}p_\nu$, but promoting the term to an operator as we did in (2.40) makes it no longer equivalent. The ambiguity makes unclear the right choice, and it also happens for the matter part [47]. As we are unable to know whether we made the right choice or not [48], we will use the named Laplace-Beltrami operator whose general form is

$$\nabla_{\mathrm{LB}} = \frac{1}{\sqrt{|g|}}\partial_\mu\left(\sqrt{|g|}g^{\mu\nu}\partial_\nu\right), \tag{2.43}$$

  which is, at least, covariant. Some studies has been done in order to avoid the operator ordering problem (e. g. see [49]), and in which, accidentally, the operator (2.43) gains importance. However, since the problem is due to the non-commutativity of the operators in QM, the expectation is that the effect of the factor ordering is only affecting the early stages of the evolution of our universe [50], where the quantum interactions are stronger.

- In Eq. (2.41) we have premiered the wave function of the universe $\Psi(h_{ij}, \{\varphi\})$ accountable for the quantum description of the whole universe in terms of the 3-metric h_{ij} and the matter fields. The infinite-dimensional space built on the metric h_{ij} and the fields $\{\varphi\}$ is called *superspace*. In general, it is simple to realise that a solution $\Psi(h_{ij}, \{\varphi\})$ in the superspace is almost impossible to be found unless we limit the complexity of the functional differential equation. Its interpretation, in spite of its given name, is not as a usual wave function since, for instance, it is not normalizable in general [51–53]. The normalizability is usually sorted out by a more suitable interpretation based on the Many-World Interpretation [54]. The predictions from CQG are then for those values around the peak of the wave function of the universe [55], and none for the oscillating or vanishing intervals. Hence, the WKB approximation or any other semiclassical approximations are suitable for the theory without loss of predictability [53, 56].

- Another physical problem is the choice of good *boundary conditions* for the wave function. In principle, what the right boundary conditions are is still in debate [57]. The proposals have been many. The most used ones are: (1) the condition given by DeWitt [3] to avoid the initial singularity imposing the vanishment of the wave function, (2) the no-boundary proposal by Hartle and Hawking [38] in which space and time do not exist at the origin, and (3) the tunneling proposal, which demands just outgoing waves for the late state of the universe [58]. We will specify our choice later on.

- The wave function of the universe represents a system for which time is not well defined as in QM or Classical Mechanics [59–61]. This is known as the *problem of time* in CQG. Time is an external variable in classical theories, serving as a parametrization of any motion. The system a universal wave function represents evolves according to the Wheeler-DeWitt equation throughout the superspace. The notion of an external time is missing here since, in principle[1], the observers sensible to such time are not there. Recovering the usual notion of time is done in a semiclassical way [56, 62–64]. The scale factor is thus recognized as a kind of parametrization of

[1] No religious affront is intended.

time, which leads to physical questions since, for some models, the parametrization is not well-defined, not being an isomorphism and, for instance, no difference between Big Bang and Big Crunch can be inferred.

All the problems here listed have many more consequences and lead to physical and philosophical discussions which we, and anyone, avoid somehow for the sake of researching in QC. The mathematical treatment of CQG may be simpler for simplified superspaces, but there is no way to get rid of those conceptual problems introduced above.

2.2.2 The Minisuperspace of FLRW Geometries

The most brutal, though intelligent, simplification one can perform on the infinite-dimensional superspace is the reduction into a two-dimensional superspace [42], then called *minisuperspace*, made of the unique coordinate of homogeneous and isotropic universes [25], the scale factor $a(t)$, and a single scalar field $\phi(t)$. The procedure here on is analogous to the one we did throughout Section 2.2.1.

The metric (2.32) shows the properties of homogeneity and isotropy up under the choice of a certain foliation for which the shift vector N^a vanishes. Thus, for a flat universe, $\mathtt{k} = 0$, the FLRW metric (2.7) in terms of the cosmological time t is

$$\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\left[\mathrm{d}r^2 + r^2\mathrm{d}\Omega_2^2\right], \tag{2.44}$$

where we have chosen spherical coordinates, and $\mathrm{d}\Omega_2^2$ is the differential section of the solid angle.

Using the metric (2.44) into the Einstein-Hilbert action (2.6) with the GHY term yields [42]

$$S_{\mathrm{EH}} = \int \mathrm{d}tL = \frac{1}{2}\int \mathrm{d}tN\left[-\frac{a\dot{a}^2}{N^2} + a\mathtt{k} - \frac{\Lambda a^3}{3} + \frac{a^3\dot{\phi}^2}{N^2} - 2a^3V(\phi)\right], \tag{2.45}$$

where we integrated over d^3x, we defined $G = 3\pi/2$, and we rescaled the field $\phi \to \phi/(\sqrt{2}\pi)$, and its potential $V \to V/(2\pi^2)$.

The equation of motion for N, which is non-dynamical since $p_N = \partial_{\dot{N}} L = 0$, yields

$$\dot{p}_N = \frac{\partial L}{\partial N} = \frac{1}{2}\left[\frac{a\dot{a}^2}{N^2} - \frac{a^3\dot{\phi}^2}{N^2} + a\mathtt{k} - \frac{\Lambda a^3}{3} - 2a^3 V(\phi)\right] = 0, \tag{2.46}$$

which is a constraint we recognize as the Friedmann equation

$$H^2 = \frac{\Lambda}{3} - \frac{\mathtt{k}}{a^2} + 2\rho(\phi), \tag{2.47}$$

after introducing the relation for the density in (2.12) and choosing $N = 1$, hence our time variable is then recognized as the cosmic time t. With the same choice for N, the equation of motion for a is

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} - [\rho(\phi) + 3p(\phi)], \tag{2.48}$$

where we used the relation for the pressure in (2.12) and Eq. (2.47) to simplify it. This is the second Friedmann equation.

The Legendre transformation of the Lagrangian (2.45) makes the Hamiltonian to be

$$H = \frac{N}{2}\left[-\frac{p_a^2}{a} + \frac{p_\phi^2}{a^3} - a\mathtt{k} + \frac{\Lambda a^3}{3} + 2a^3 V(\phi)\right] = \frac{N}{2}\left[-G^{ab}p_a p_b - a\mathtt{k} + \frac{\Lambda a^3}{3} + 2a^3 V(\phi)\right], \tag{2.49}$$

where the canonical momenta are

$$p_a = -\frac{a\dot{a}}{N}, \qquad p_\phi = \frac{a^3\dot{\phi}}{N}, \qquad p_N = 0. \tag{2.50}$$

The quantization of the momenta is chosen to keep covariance having the form of the Laplace-Beltrami operator (2.43) where $G_{ab} = \mathrm{diag}(-a, a^3)$ is the DeWitt's metric we use to define it, and thus

$$\nabla_{\mathrm{LB}} = -\frac{1}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) + \frac{1}{a^3}\frac{\partial^2}{\partial \phi^2}. \tag{2.51}$$

That way, the simplified version of the WDW equation (2.41) for the minisuperspace, given for homogeneous and isotropic universes, is then

$$\left[\frac{1}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{1}{a^3}\frac{\partial^2}{\partial \phi^2} - a\mathtt{k} + \frac{\Lambda a^3}{3} + 2a^3 V(\phi)\right]\Psi(a,\phi) = 0. \tag{2.52}$$

Before commenting about this equation, lets change it a bit for our convenience. Usually, a more comfortable expression of Eq. (2.52) is obtained with the well-behaved parametrization of the scale factor $\alpha = \ln(a)$:

$$\left\{ \frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} - \mathrm{e}^{4\alpha}\mathbf{k} + \mathrm{e}^{6\alpha}\left[\frac{\Lambda}{3} + 2V(\phi)\right]\right\}\Psi(\alpha,\phi) = 0. \tag{2.53}$$

It is much convenient since it is now a KG-like equation, it is, analogous to Eq. (2.18), and because $\alpha \in]-\infty, \infty[$ and not as $a \in [0, \infty[$, which is a more suitable option to define spacetime variables in a quantum theory. The Laplace-Beltrami operator (2.51) can now be seen as a box operator, and the third and fourth term of Eq. (2.53) the mass of $\Psi(\alpha, \phi)$:

$$m^2(\alpha,\phi) = -\mathrm{e}^{4\alpha}\mathbf{k} + \mathrm{e}^{6\alpha}\left[\frac{\Lambda}{3} + 2V(\phi)\right]. \tag{2.54}$$

Besides, trying to get a perfect comparison with Eq. (2.18), there is an ambiguity about what variable is playing the role of time or space. In general, α is taken to be the variable playing the role of time for many reasons. Classically, the scale factor a is an explicit function of time, and so is α, which is for the universe we inhabit, apparently monotonic [25]. This leads to problems with the interpretation of the wave function $\Psi(\alpha, \phi)$ when $a(t)$ is not monotonic [65].

For instance, let us consider a closed universe with a massless free scalar field $V(\phi) = 0$. The Friedmann equation (2.47) is simply

$$H^2 = \dot{\phi}^2 - \frac{1}{a^2}. \tag{2.55}$$

The momentum associated with the scalar field in (2.50) is then constant, and so

$$\dot{\phi} = \frac{k}{a^3}, \qquad k \in \mathbb{R}. \tag{2.56}$$

Replacing $\dot{\phi}$ into (2.55), the scale factor is implicitly found as a function of time like

$$t - t_o = \int \frac{a^2 \mathrm{d}a}{\sqrt{k^2 - a^4}}, \tag{2.57}$$

whose shape $a(t)$ is very similar to a simple semicircle into the interval $[0, a_{\max}]$, where $a_{\max} = \sqrt{|k|}$ is the maximum size the universe gets. Furthermore, the trajectories in the phase space are

$$\phi(a) = \phi_o \pm \frac{1}{2}\mathrm{arccosh}\left(\frac{k}{a^2}\right). \tag{2.58}$$

This example shows up clearly that there is a problem with the notion of time in QC, or the interpretation of the scale factor as a good time variable. The wave function

$\Psi(a,\phi)$ will not be different for expanding and contracting phases of the evolution since it is not dependent on $\dot{a}$ but simply on a. The example is posed in Ref. [65], where the analysis of wave packets drops the conclusion that at the returning point $a_{\max}$, the wave packets interfere constructively, which means that the quantum behavior is remarkable. The contracting branch must be seen as a wave packet in the minisuperspace coming back in time from the Big Crunch singularity, where the classical universe collapses.

In general, we will work with non-interacting massless scalar fields unless we specify something else. The potential of the scalar field is the set as $V(\phi) = 0$, and therefore the WDW equation (2.53) is separable since $m(\alpha,\phi) = m(\alpha)$. We split the differential equation such that

$$-\frac{\partial^2}{\partial\phi^2}\Psi(\alpha,\phi) = E_\phi\Psi(\alpha,\phi), \qquad \left[\frac{\partial^2}{\partial\alpha^2} - \mathrm{e}^{4\alpha}\mathbf{k} + \mathrm{e}^{6\alpha}\frac{\Lambda}{3}\right]\Psi(\alpha,\phi) = E_\alpha\Psi(\alpha,\phi), \tag{2.59}$$

where $E_\alpha + E_\phi = 0$. Now, expanding the wave function in its Fourier modes

$$\Psi(\alpha,\phi) = \int_{\mathbb{R}} \mathrm{d}k A(k)\varphi_k(\alpha)\chi_k(\phi), \tag{2.60}$$

where $A(k)$ is the distribution function of the modes k of the scalar field, one solves from (2.59) the part corresponding to the matter field

$$\chi_k(\phi) = \mathrm{e}^{\pm\mathrm{i}k\phi}, \tag{2.61}$$

and uses it to write the WDW equation for $\varphi_k(\alpha)$ as

$$\left[\frac{\partial^2}{\partial\alpha^2} + k^2 + m^2(\alpha)\right]\varphi_k(\alpha) = 0. \tag{2.62}$$

The solutions to this differential equation, in most of the cases, consist of Bessel functions of some kind[2]. The boundary conditions for the wave functions here are found in the limit $\alpha \to -\infty$, where the WDW equation (2.62) is approximated to a free wave equation after $m(\alpha) \to 0$ rapidly. In such asymptotic region, we impose the usual condition (also used for cosmological perturbations of the inflaton [68]) such that only wave moving forward like [35, 69]

$$\varphi_k(\alpha) = \frac{1}{\sqrt{2k}}\mathrm{e}^{-\mathrm{i}k\alpha} \tag{2.63}$$

[2]For definitions and properties of special functions I recommend to check Ref. [66] which is the updated version of the renowned book in Ref. [67]. In special, I recommend the online version of it.

are considered. Here, when we try to obtain boundary conditions for the wave function, we can see the importance of the shape of Eq. (2.53) compared to (2.52).

Another condition which is usually imposed to some models is that the wave function of the universe is vanishing as $\alpha \to \infty$. Such condition can be understood for closed universes like the one I wrote about above if one treats the wave function as a source of probability and wants to suppress the probability of infinitely expanded universes. For instance, the wave function for a closed universe with a massless scalar field fulfilling this condition goes like [42]

$$\varphi_k(\alpha) \propto \mathcal{K}_{\frac{ik}{2}}\left(\frac{e^{2\alpha}}{2}\right), \tag{2.64}$$

where $\mathcal{K}_\nu(z)$ is the modified Bessel function of the second kind. However, it does not fulfill the condition (2.63) when $\alpha \to -\infty$, since (2.64) is real everywhere. This condition will be dropped for us since it seems a naive condition in general as it does not make any sense for open universes, but the most important reason will be explained in Section 2.3.

It will be important to notice that, in principle, the distribution function $A(k)$ in Eq. (2.60) is not subjected to any constraint, and it is a complex function of the modes k of the scalar field. Recovering the wave function $\Psi(\alpha, \phi)$ requires knowing the form of $A(k)$. I will find strong constraints and select a very natural function of the modes for it in Chapter 3.

2.3 Third Quantization and a Pair of Universes

2.3.1 A Quantum Field Theory of Universes

Up to now, the wave function of the universe is similar to a classical field $\Psi(\vec{r}, t)$ representing a complex scalar field as in Eq. (2.18). The discussion of whether it is scalar, spinor [70], or any other kind [71] of wave function is not new, and it is still under review [72]. Nevertheless, it is common in the literature to consider it a complex scalar field and that is what we will do.

Third Quantization (3rdQ) of CQG is a longstanding paradigm which follows the same scheme we did in Section 2.1.3. Starting from the WDW equation (2.52), in the 3rdQ picture the wave function of the universe is promoted to an operator acting on vectors of a Fock space $\{|n_i\rangle\}$ and thus we create a quantum field theory of universes. Such vectors define states of excitation of the universes, or equivalently number of universes, in the mode k_i. The pioneer work on the topic is said to be Ref. [11], and applications [73–79] and specialized reviews [80] have been published until today.

The WDW equation (2.53) is derived from the action

$$S_{3\mathrm{Q}} = \frac{1}{2}\int \mathrm{d}\alpha \mathrm{d}\phi \left[-\eta^{\mu\nu}\partial_\mu \Psi^\dagger \partial_\nu \Psi - m^2(\alpha,\phi)\Psi^\dagger\Psi\right], \tag{2.65}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1,+1)$ and $\Psi^\dagger$ is representing an antiparticle of Ψ. It is analogous to the action (2.21) for complex scalar fields. An expansion like (2.25) represents any state in terms of certain annihilation and creation operators b_k and $b_k^\dagger$, respectively, which are responsible for the mode k of the state.

The solutions of the WDW equation (2.62) are dependent on k, which is unique for each solution, and also on α. It makes the solutions to be different for the *in* and *out* regions, for which $\alpha \to -\infty$ and $\alpha \to \infty$, respectively. Examples are given in Refs. [76, 77, 81, 82], where they calculated both solutions, Ψ_k^{in} and Ψ_k^{out}, and expanded in terms of the ladder operators C_{in} and C_{out}:

$$\Psi = \int \mathrm{d}k \left[C_{\mathrm{in}}\Psi_k^{\mathrm{in}} + C_{\mathrm{in}}^\dagger \Psi_k^{\mathrm{in},*}\right] = \int \mathrm{d}k \left[C_{\mathrm{out}}\Psi_k^{\mathrm{out}} + C_{\mathrm{out}}^\dagger \Psi_k^{\mathrm{out},*}\right]. \tag{2.66}$$

Whatever was the definition of the vacuum at the in-region, the result of the expectation value for each mode k of the number operator as in Eq. (2.30), where $\left|\bar{0}\right\rangle$ is the vacuum described by the Fock states of the out-region, is a kind of Planckian distribution. Hence, and infinite amount of universes with a specific spectrum are created in the multiverse, analogously to the Unruh effect [83–85].

The interesting characteristic of this special picture of the multiverse is that the description of the interactions among the unknown amount of universes could be thought to be quite small and described by the Lagrangian in Eq. (2.65) adding any kind of interaction terms with their own coupling constants. Seeing the 3rdQ picture as a quantum field theory of universes is rather natural. The creation of baby universes from parent universes subjected to a necessary change on its topology is easily explained this way [74, 86–88], and the most probable interaction must be the pair creation after the decay of a single parent universe, whose QFT counterpart here is therefore the Schwinger effect [89].

2.3.2 A Pair of Universes and their Entanglement

2.3.2.1 A Universe and its Antiuniverse

At a singular point in the minisuperspace, a pair of universes can be created as a particle and an antiparticle, and thus we can name them as universe and antiuniverse. Given the 0-spin nature we gifted the universes in our paradigm, each complex scalar function which describes a singe universe of a pair must be the complex conjugated of its partner one. The initial state from where they started is denoted by $|00\rangle$, following the notation $\left|U^{\dagger}U\right\rangle = \left|U^{\dagger}\right\rangle |U\rangle$, which is the combined quantum state of both universes. That is the vacuum state where neither time nor space existed.

Here, the vector $\left|U^{\dagger}\right\rangle$ is the quantum state of a contracting branch of the universes, and $|U\rangle$ is an expanding one. However, time is split in two directions for both universes and, as seen for themselves, they are both expanding. A pictorial representation of the pair is shown in Figure 2.2. Those states could have also been denoted by $|U^{+}\rangle$ and $|U^{-}\rangle$ for comparison with pair creation.

It is interesting to note that both are twins, but not necessarily identical or copies. Each universe, after its creation, is subjected to the quantum rules which are purely probabilistic. The fate of each one, as for example the appearance and distribution of the large scale structure [90], will be mostly defined by the quantum fluctuations during their early times, which is very unlikely that both universes replicate them ideally.

The representation in which the wave functions of each universe are obtained from the WDW equation (2.52) or any equivalent, is usually called *diagonal* since the

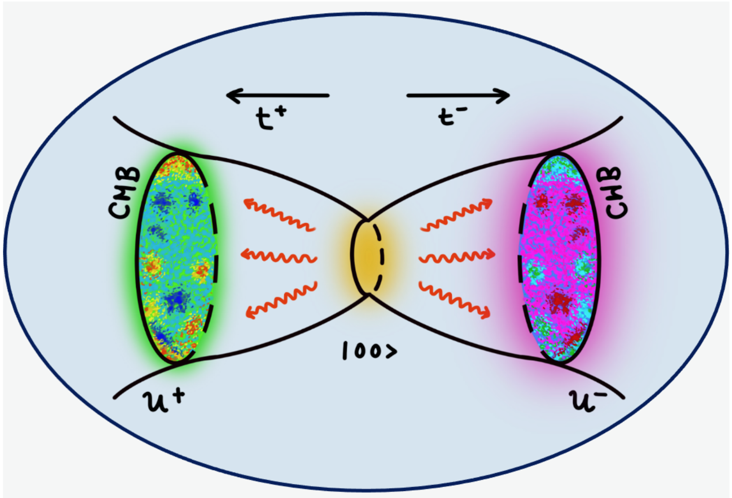

Figure 2.2: Pictorial representation of the pair, universe and antiuniverse, acting like particle and antiparticle, respectively.

Hamiltonian which describes the system is separable

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2, \tag{2.67}$$

where $\mathcal{H}_i$ is each individual Hamiltonian of the pair. Any other representation of the system would involve non-separable Hamiltonians

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_{\text{int}}, \tag{2.68}$$

where $\mathcal{H}_{\text{int}}$ is the non-diagonal part of the representation, which is responsible for an interaction between them.

Even though the universes are described by complex scalar fields, it is still possible that they are entangled via the mixture of states given by the non-diagonal representation. That entanglement cannot be given any reasonable sense, unless we set a realistic representation like the invariant one.

2.3.2.2 The Invariant Vacuum

It is clear after the discussion in Section 2.3.1 that the state $|00\rangle$ of the pair will not be the vacuum for the rest of its evolution since their wave functions depend on the individual evolution of the universes. At any other point of the evolution, the number of universes is expected to change with a certain spectrum of modes. The pair will no longer be a pair of well-described modes, but an infinite combination of states $|n_- n_+\rangle$, where $|n_i\rangle$ is the excitation mode of each universe. Therefore, even if at the calculational level the diagonal representation is simpler, it does not seem to be adequate so that the pair remains described by the same state. It is then convenient to introduce a non-diagonal representation in which the number operator is constant for the entire evolution of the pair, called the *invariant representation*.

The invariant representation was first introduced by Lewis and Riesenfeld [91] in order to study the time-dependent quantum harmonic oscillator

$$\left[\frac{\partial^2}{\partial t^2} + \omega^2(t)\right] \Psi(t) = 0, \tag{2.69}$$

where $\omega(t)$ is the time-dependent frequency. They found the transformations from the diagonal representation to the invariant representation in which the number operator is invariant. Afterwards, the formalism was improved [92–96] into a more tractable form and reviewed for cosmological applications [97].

Concerning the cosmological applications, one is able to apply this formalism for certain WDW equations which are comparable to a time-dependent harmonic oscillator identifying

$$\omega^2(\alpha, \phi) = -\frac{\partial^2}{\partial \phi^2} + m^2(\alpha, \phi), \tag{2.70}$$

after comparison of Eqs. (2.53) and (2.69). A simple inspection of Eq. (2.53) shows that the simplest case in which we get a time-dependent harmonic oscillator-like equation is when there is no scalar field at all and only gravity is left, or (anti-)de Sitter spacetimes, where there is a scalar field whose kinetic energy vanishes, or equivalently, a cosmological constant Λ is different from zero. As we infer after the inspection, in general, it is not possible to apply the formalism as it was given to us. The usage of this formalism for Cosmology is not new [98], and it will be very useful for what is going to follow in next sections. The application for a non-interacting massless scalar field will be given in Chapter 3.

Let us denote as $b_\pm$ and $b_\pm^\dagger$ the annihilation and creation operators, respectively, in the diagonal representation, where the $+$ labels the universe and the $-$ labels the antiuniverse branch. Besides, $c_\pm$ and $c_\pm^\dagger$ are the ones associated with the invariant representation, respectively. The transformation between both representations is then

$$\hat{c}_- = \alpha_B \hat{b}_- - \beta_B \hat{b}_+^\dagger, \qquad \hat{c}_-^\dagger = \alpha_B^* \hat{b}_-^\dagger - \beta_B^* \hat{b}_+, \tag{2.71}$$

where α_B and β_B are the Bogoliubov coefficients of the transformation. Defining the function[3]

$$R = \sqrt{\Psi_1^2 + \Psi_2^2}, \tag{2.72}$$

where Ψ_i are two linearly independent real solutions to the time-dependent harmonic oscillator, those Bogoliubov coefficients are found to be written as

$$\alpha_B = \frac{1}{2}\left[\frac{1}{R\sqrt{\omega}} + R\sqrt{\omega} - \mathrm{i}\frac{\dot{R}}{\sqrt{\omega}}\right], \qquad \beta_B = -\frac{1}{2}\left[\frac{1}{R\sqrt{\omega}} - R\sqrt{\omega} - \mathrm{i}\frac{\dot{R}}{\sqrt{\omega}}\right]. \tag{2.73}$$

In such a new representation, the vacuum state will remain constant at any time, and it is different from the vacuum described by the diagonal representation at any time, except at the precise moment of the creation, where both vacuum coincide. That is why we will denote the same vacuum state as $|00\rangle_d$ and $|00\rangle_i$, being described by the diagonal and the invariant representations, respectively. Applying it to a proper WDW equation, the invariant representation is thus a very natural representation in which the pair is unchanged. The selection of it makes the calculation of the entanglement entropy almost straightforward.

2.3.2.3 The Entanglement Entropy

The entanglement entropy (EE) of a bipartite system like ours is customarily taken as the von Neumann entropy [99–101]

$$S^{\mathrm{N}} = -\operatorname{Tr}\{\rho_A \ln(\rho_A)\}, \tag{2.74}$$

[3] Generalization of R that may be useful for different applications can be found in Refs. [93, 94]. The function is

$$R = \sqrt{c_1\Psi_1^2 + c_2\Psi_1\Psi_2 + c_3\Psi_2^2},$$

with the constraint $W^{-2} = c_1c_3 - c_2^2/4$, where W is the Wronskian that must be constant. It is enough to set $c_2 = 0$ and $c_1, c_3 = 1$, which is our case.

where ρ_A is the reduced density matrix

$$\rho_A = \mathrm{Tr}_B(\rho_{AB}) = \mathrm{Tr}_B\left(\sum_{ijkl} c_{ijkl} \left|a_i\right\rangle \left\langle a_j\right| \otimes \left|b_k\right\rangle \left\langle b_l\right|\right) = \sum_{ijkl} c_{ijkl} \left\langle b_l|b_k\right\rangle \left|a_i\right\rangle \left\langle a_j\right|, \quad (2.75)$$

of the bipartite system defined by the density matrix ρ_{AB}, where $\{|a_m\rangle\}$ and $\{|b_n\rangle\}$ are the vectors of the Hilbert space associated with A and B, respectively. This entropy is bounded in the interval $[0, \log(\dim(\mathfrak{H}))]$, where $\mathfrak{H}$ is the Hilbert space in which it is described. The maximum value of the EE is characteristic of maximally entangled systems. The simplest example is a system of two qubits, whose maximum value is ln(2), since the Hilbert space $\mathfrak{H}$ is two-dimensional, and its canonical basis is $\{|\uparrow\rangle, |\downarrow\rangle\}$. Therefore, and since the entanglement is a unique property of the quantum world, the EE is said to be a measurement of the *quantumness* of a system. Another definitions of the entropy, and hence indicators of the quantumness, are the Rènyi [102, 103] and Tsallis [104, 105] entropies, which extend the von Neumann one (2.74) like

$$S_q^{\mathrm{T}} = \frac{1}{1-q}\left[\mathrm{Tr}(\rho^q) - 1\right], \qquad S_q^{\mathrm{R}} = \frac{1}{1-q}\left[\mathrm{Tr}(\rho^q)\right], \quad (2.76)$$

respectively, where q is a parameter. These entropies are said to be generalizations because one recovers von Neumann entropy (2.74) taking the limit $q \to 1$ [104, 106].

In order to find the EE of the pair of universes, one is compelled to use the invariant representation in which the vacuum $\left|00\right\rangle_i$ is invariant at any time. Thus, the pair is described by the constant density operator $\rho = \left|00\right\rangle_i \left\langle 00\right|$. It can be expressed in terms of the diagonal states $\left|n_- n_+\right\rangle_d$ using the relation [69, 107–110]

$$\left|00\right\rangle_i = \frac{1}{|\alpha_B|}\sum_{n=0}^{\infty}\left(\frac{|\beta_B|}{|\alpha_B|}\right)^n \left|n_- n_+\right\rangle_d. \quad (2.77)$$

The reduced density matrix defined as in (2.75) is the density matrix for each single universe of the pair. For example, taking the partial trace of the universe whose states are $|n_+\rangle$, it yields

$$\rho_- = \sum_{n=0}^{\infty} \left\langle n_+|\rho|n_+\right\rangle_d \propto \sum_{n=0}^{\infty} \exp\left\{\frac{\omega(\alpha)}{T(\alpha)}\left(n + \frac{1}{2}\right)\right\} \left|n_-\right\rangle_d \left\langle n_-\right|, \quad (2.78)$$

where $\omega(\alpha)$ is the associated frequency to the time-dependent harmonic oscillator-like equation (2.69), we have defined the temperature of entanglement

$$T(\alpha) = \frac{\omega(\alpha)}{2\ln\left[\coth(r)\right]}, \quad (2.79)$$

and the parameter

$$Q = \tanh(r) = \frac{|\beta_B|}{|\alpha_B|}, \tag{2.80}$$

which can be found to be bounded in the interval $[0, 1]$ attending to the condition (2.27). As a conclusion, it was stated in Ref. [111] that the temperature of entanglement and the parameter Q could also be a good alternative to quantify the quantumness of a system. An analysis of such statement is going to be done throughout Chapter 3.

The EE, which is the von Neumann entropy (2.74), comes directly from Eq. (2.78) as [112, 113]

$$S^{\mathrm{N}}(\alpha) = \cosh^2(r) \ln\left[\cosh^2(r)\right] - \sinh^2(r) \ln\left[\sinh^2(r)\right]. \tag{2.81}$$

This expression is bounded in $\mathbb{R}^{+*}$ as r is also in the same set. Also, I expressed the Tsallis and Rènyi entropies (2.76) in terms of r as

$$S_q^{\mathrm{T}}(\alpha) = \frac{1}{1-q}\left[\frac{\operatorname{sech}^q(r)}{1-\tanh^{2q}(r)} - 1\right], \qquad S_q^{\mathrm{R}}(\alpha) = \frac{1}{1-q}\left[\frac{\operatorname{sech}^q(r)}{1-\tanh^{2q}(r)}\right], \tag{2.82}$$

to compare it with the von Neumann entropy in the following.

In order to maintain the idea of the pair as particle and antiparticle whose wave functions are complex conjugated, we cannot choose, for example, the solution (2.64) as a good one since it is real. Besides, such a solution is given by the boundary condition of a vanishing wave function of the universe as α approaches infinity, which yields a single solution. It makes impossible to consider such a boundary condition if our intention is to find the EE, because we demand two linearly independent solutions to use for the function R in Eq. (2.72). A more natural boundary condition is the one in Eq. (2.63) which sets the universes starting in the Bunch-Davies vacuum.

3

Quantum Entanglement of a Pair of Universes

Here I present the results to which I arrived based on the foundations showed along Chapter 2. In general, we will focus our attention on the analysis of a pair of universes with a massless scalar field born in the lap of 3rdQ of CQG. There is a variety of results we will need to go on into the observational work done in Chapter 4. Those are the most important outcomes. However, the theoretical results I get here are essential for the observational part of this text.

As theoretical physicists, we care about the behaviour of whatever the dynamics of a system can be. It is, we are not going to focus just on realistic scenarios whence we may be missing some important theoretical understanding of the Physics underneath. It will be the aim of Chapter 4 to get closer to reality and check the falsifiability of the theory and the assumptions we work with and conclusions we get in the present Chapter.

Firstly, I will treat some mathematical details needed in order to find the EE of the pair of universes in a consistent manner. They will have a very important physical consequences which, secondly, we will use and check along the sections in many different models of FLRW universes which include models with standard and exotic singularities.

3.1 Existence of Solutions for a Pair of Universes

First of all, a question arises about the suitability of the pair production in the 3rdQ picture. In a perfect analogy to QFT, we demand that the wave function for each universe of the pair is complex conjugated to the other one. But, does the WDW equation admit two solutions with such a condition? In principle, I will assume that the WDW equation accept a separable solution in Fourier modes with a certain distribution function $A(k)$ like (2.60), where the matter field function is (2.61), and thus the gravitational part fulfills Eq. (2.62).

Eq. (2.62) can be written in terms of the scale factor $a = \mathrm{e}^{\alpha}$, like

$$\left\{a^2 \frac{\mathrm{d}^2}{\mathrm{d}a^2} + a\frac{\mathrm{d}}{\mathrm{d}a} + \left[k^2 + m^2(a)\right]\right\} \varphi_k(a) = 0, \tag{3.1}$$

which is a second-order ordinary differential equation. Assuming that the expression into the square brackets is analytical, it is necessary that $m^2(a)$ is also analytical, such that

$$m^2(a) = \sum_{n=0}^{\infty} m_n a^n, \tag{3.2}$$

where m_n are constants with at least one of them different from zero. If that is the case, one can use the Frobenius method for differential equations [66] to solve (3.1). The indicial equation I get is

$$R(R-1) + R + k^2 + m_o = 0, \tag{3.3}$$

from where I find

$$R_{1,2} = \pm \mathrm{i}k, \tag{3.4}$$

knowing that m_o is vanishing, in general, after inspection of Eq. (2.59). It implies that the solutions $\varphi_k(a)$ can be obtained as

$$\varphi_k^{(1)}(a) = a^{R_1} \sum_{n=0}^{\infty} b_n(R_1) a^n, \qquad \varphi_k^{(2)}(a) = a^{R_2} \sum_{n=0}^{\infty} c_n(R_2) a^n, \tag{3.5}$$

where b_n and c_n are constant that must be found by substitution into Eq. (3.1). Regardless of any precise expression of $m^2(a)$ and the constants b_n and c_n, we can see from (3.5) that both solutions $\varphi^{(1,2)}(a)$, and hence $\varphi^{(1,2)}(\alpha)$, are complex conjugated since R_1 is the complex conjugated of R_2. Those solutions are going to be equivalent under the transformations

$$k \mapsto -k, \qquad \text{or equivalently,} \qquad \mathrm{i} \mapsto -\mathrm{i}, \tag{3.6}$$

since both change the solutions (3.4) to the indicial equation one into the another.

Building the total wave function $\Psi(\alpha, \phi)$ of the particle-like universe (2.60) is just a matter of choosing one of those two functions $\varphi^{(1,2)}(\alpha)$, any sign for the matter part (2.61) and any complex function $A(k)$. On the other hand, to get the antiparticle-like wave function, one simply uses the complex conjugated functions of the ones we chose. Therefore, there is always a pair of solutions which can be used as wave functions of the universe and the antiuniverse of the pair, and which are in agreement with the picture offered by 3rdQ formalism.

3.2 The Distribution Function $A(k)$

Up to now, I have not restricted the complex distribution function $A(k)$ in any sense. A property of it comes out as a side effect of restoring the matter-antimatter asymmetry of the universe via 3rdQ [114]. The matter is described by positive modes and antimatter by negative ones, but a general wave function is a mix of them. The solutions (2.60) and its complex conjugated are integrated over all $k \in \mathbb{R}$, and fortunately they are not unique. Another integration interval would yield two different complex conjugated solutions.

For instance, let me consider the solutions

$$\Psi^{(1)}(\alpha, \phi) = \int_0^{\infty} \mathrm{d}k A(k) \mathrm{e}^{\mathrm{i}k\phi} \varphi_k^{(1)}(\alpha), \qquad \Psi^{(2)}(\alpha, \phi) = \int_{-\infty}^{0} \mathrm{d}k A(k) \mathrm{e}^{\mathrm{i}k\phi} \varphi_k^{(1)}(\alpha), \tag{3.7}$$

which are just two wave functions obtained from different integration intervals of a certain solution of Eq. (2.60). My intention here is to bestow the positive modes on a universe and the negative ones on an antiuniverse. As we concluded in Section 3.1, performing a complex conjugation or the change $k \mapsto -k$ is equivalent for the solutions of Eq. (2.62), and it also happens to the matter part (2.61). A fast calculation shows that

$$\Psi^{(2)}(\alpha,\phi) = \int_{-\infty}^{0} \mathrm{d}k A(k) \mathrm{e}^{-\mathrm{i}k\phi} \left[\varphi_k^{(1)}(\alpha)\right]^*, \tag{3.8}$$

from where I conclude that $A(k)$ must be symmetric around $k = 0$ in order to get the condition we expected $\Psi^{(1)}(\alpha,\phi) = \left[\Psi^{(2)}(\alpha,\phi)\right]^*$. This also ensures the conservation of the energy of the system since the same amount of particles are in both universes whose energies are also the same.

From now on, I will only consider $A(k)$ to be a Gaussian function around $k = 0$ with σ as its standard deviation

$$A_\sigma(k) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{k^2}{2\sigma^2}}, \tag{3.9}$$

which was also used in Ref. [65], but there was not a physical reason to use it as in here.

3.3 Calculation of the Entanglement Entropy

The algorithm to find the EE for any of the models we will use is as follows. As discussed in Section 2.3.2.2, there are only a few cosmological models which can be compared to a time-dependent harmonic oscillator such that there exists an invariant representation that can be found with the Lewis-Riesenfeld algorithm. The case under study includes non-interacting massless scalar fields, that is $V(\phi) = 0$. In such a case, the frequency (2.70) is just

$$\omega^2(\alpha,\phi) = -\frac{\partial^2}{\partial\phi^2} + m^2(\alpha) = -\frac{\partial^2}{\partial\phi^2} + \mathrm{e}^{6\alpha}\frac{\Lambda}{3} - \mathrm{e}^{4\alpha}\mathtt{k}. \tag{3.10}$$

This operator can be rewritten as

$$\omega^2(\alpha) = E_\phi + m^2(\alpha), \tag{3.11}$$

using the expression (2.59), which is an expression dependent only on α and where E_ϕ is a constant to be fixed, as usual, to a positive number. It makes it usable for the Lewis-Riesenfeld algorithm.

In order to apply it, the expression (2.72) demands two real wave functions. We will take the real and the imaginary part of any of the solutions (3.7) or (3.8), which are also solutions of the WDW equation (2.62). The important step is to obtain conjugated solutions $\varphi_k^{(1,2)}(\alpha)$. Since the boundary condition we apply is the one in Eq. (2.63), we can be sure that the numerical method to solve Eq. (2.62) will yield two solutions which are complex conjugated. Therefore, it is only necessary to calculate one of them – its real and imaginary part. Then, we need to set a sign for the matter part (2.61) and the standard deviation σ of the distribution function $A_\sigma(k)$ in Eq. (3.9). With all those bricks, we build the total wave function (2.60) whose real and imaginary parts will be $\Psi^{(1)}(\alpha)$ and $\Psi^{(2)}(\alpha)$, and thus, we get R in Eq. (2.72).

Once we know everything into the expressions of the Bogoliubov coefficients (2.73), we can use them to find the EE (2.81) using the relation (2.80), and also the Tsallis and Rènyi entropies (2.82). As mentioned before, we will check if the temperature (2.79) and the parameter Q (2.80) are good measurements of the quantumness of a system with some of our models.

3.4 Quantum Entanglement and the Classical Phase Space

An unexpected property of the EE (2.81) was found including the expressions of the Bogoliubov coefficients (2.73) into it. Particularly, I get

$$\cosh^2(r) = \frac{1}{1-q^2} = \begin{cases} \dfrac{\left(\frac{1}{R} + R|\omega|\right)^2 + \dot{R}^2}{4|\omega|}, & \text{if} \quad \omega^2 \geq 0 \\ -\dfrac{\frac{1}{R^2} + \left(R|\omega| - \dot{R}^2\right)^2}{4|\omega| R\dot{R}}, & \text{if} \quad \omega^2 < 0 \end{cases}, \tag{3.12}$$

which is of relevant importance to infer the form of the EE.

As we know from Classical Mechanics, the turning points of a system are obtained as those for which the kinetic energy vanishes and the potential energy arrives at a maximum, and it is also true for the expectation values of quantum systems. In our case, the potential energy of the WDW equation (2.53) is given by Eq. (3.11), whose right hand side can be positive or negative. There are points along the evolution of the universe where the frequency vanishes. At those points, the universe arrives to a turning point, like a classical maximum or a minimum. The EE at the turning points is divergent since Eq. (3.12) diverges when $\omega^2(\alpha) = 0$.

Furthermore, the phase space is then divided in two regions classically: where the system can be found because it is classically allowed, or where it cannot be because it is forbidden. Quantumly, we have found that when $\omega^2(\alpha) > 0$, the EE is a positive real, and when $\omega^2(\alpha) < 0$, the value of $\cosh^2(r)$ is negative, and therefore the EE (2.81) is not real, since $\mathrm{Ln}(-|x|) = \ln(x) + \mathrm{i}\pi$ [115]. I can summarise it as:

$$S^{\mathrm{N}}(\alpha) = \begin{cases} \mathbb{R}^{+*}, & \text{if} \quad \omega^2 > 0 \qquad \text{(Classically Allowed)} \\ \infty, & \text{if} \quad \omega^2 = 0 \qquad \text{(Turning Point)} \\ \mathbb{R}^{+*} + \mathrm{i}\pi, & \text{if} \quad \omega^2 < 0 \qquad \text{(Classically Forbidden)} \end{cases} . \tag{3.13}$$

This classification is really unexpected since the EE is a purely quantum property of a system. Nevertheless, here it appears to offer the same classification as the classical logic. The turning points also emerge from the analysis of the EE even when, typically, every classical barrier, like a finite potential barrier, does not have the same behaviour in quantum regimes, so they stop behaving like barriers, but damping regions. It is even more unexpected when one realises that the theory does not include any process of decoherence [116].

3.5 A General Two-Dimensional Minisuperspace

As the first model, I consider a closed universe without cosmological constant filled with a non-interacting massless scalar field. The WDW equation for fixed modes (2.62) is a very simple one as

$$\left[\frac{\partial^2}{\partial\alpha^2} + k^2 - \mathrm{e}^{4\alpha}\right]\varphi_k(\alpha) = 0, \tag{3.14}$$

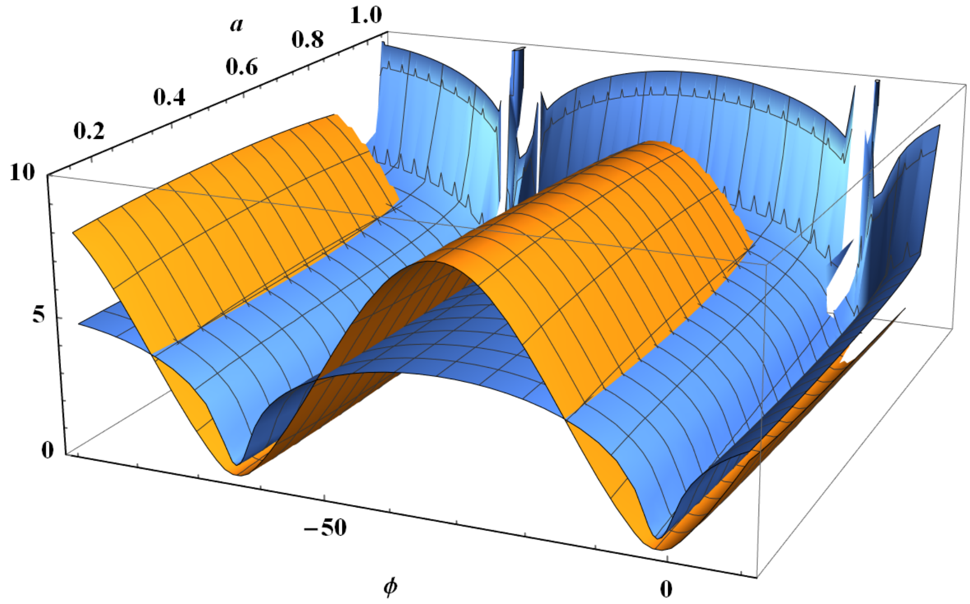

Figure 3.1: Entanglement entropy (blue surface) and the temperature of entanglement (orange surface) vs. the scale factor a and the scalar field ϕ of a pair of closed universes, with $\Lambda = 0$, $E_\phi = 1$, and $\sigma = 1$. Both are finite close to the initial singularity at $a = 0$. The entanglement entropy diverges as the universe approaches its maximum of expansion at $a = 1$. However, the temperature remains finite everywhere. Both show a wavy behaviour along the ϕ-axis. The temperature has been multiplied by 1/5 to get it into frame.

whose solutions can be found analytically as

$$\varphi_k^{(1,2)}(\alpha) \propto \mathcal{I}_{\pm \mathrm{i}k}\left(\frac{\mathrm{e}^{2\alpha}}{2}\right), \tag{3.15}$$

where $\mathcal{I}_\nu(z)$ is the modified Bessel function of the first kind. Notwithstanding, it is very unlikely that looking for solutions of (2.62) yields those ones such that they satisfy the condition (2.63). Then, in general, there is no other way but to use a numerical method including such a boundary condition.

Following the algorithm described in Section 3.3, I set $\sigma = 1$ for the distribution function (3.9), and $E_\phi = 1$. Then, the wave function is found numerically. Its real and imaginary parts are used as inputs for finding the parameter R in Eq. (2.72), and, recognizing

$$\omega^2(\alpha) = E_\phi - \mathrm{e}^{4\alpha}, \tag{3.16}$$

I directly calculate the EE at any point of the phase space. The numerical outcome is shown in Figure 3.1 as the blue surface. The temperature of entanglement (2.79) is shown in orange as well in Figure 3.1 for comparison. The EE behaves as discussed in Section 3.4. It diverges close to the maximum of expansion, which is expected at $a = 1$ for this specific model. Its unknown shape at the origin is now found to be finite even though the quantumness of the universe is understood to be high at the very beginning, which is not contradictory. Nevertheless, the EE is decreasing with the scale factor after the initial singularity. The early behaviour appears to be a very intriguing detail about the EE. The temperature of entanglement is, however, finite everywhere. Also, for both of them, there is a wavy behaviour along the ϕ-axis whose frequencies coincide. Changing the values E_ϕ and σ only changes the maximum of expansion and the frequency of the wavy form of those functions. A fast analysis of the Tsallis and Rènyi entropies (2.82) shows no much difference from the von Neumann entropy shown in the figure (a more detailed comparison will be done in Section 3.9).

Here and in the future, I get results up to errors introduced by the numerical method. In this case, the conclusion that both functions are finite around the origin is just up to the numerical value we introduced. However, comparing the result with the following ones, the precision seems to be enough and lets me infer that it is finite at the initial singularity.

3.6 Classical and Quantum WDW Equations

A Quantum Gravity theory is created with the focus on a quantum description of the universe as good as possible. Up to now, it has been the case. We have treated the variables of the minisuperspace as quantum variables whose momenta have been quantized. The quantum treatment has been extensively used [38, 53, 76, 77]. In spite of that, the ease to get solutions of different scalar field, whose potential and interactions are non-trivial, is eroded. It is the reason why the scalar field is kept classical while the scale factor is quantized in some studies [111, 117, 118]. How both treatments make a difference has been analyzed for FLRW universes and many other models [119, 120].

The WDW equation (2.52) is the Hamiltonian constraint where the scalar field has been treated as quantum. For that case, we can say that the corresponding solution is the wave function $\Psi_Q(\alpha, \phi)$, where the label Q stands for *quantum*. But, from the Hamiltonian (2.49), one can write the WDW equation just quantizing the scale factor and keeping the scalar field classical as [121]

$$\left\{\frac{\partial^2}{\partial\alpha^2} - \mathrm{e}^{4\alpha}\mathtt{k} + \mathrm{e}^{6\alpha}\left[\frac{\Lambda}{3} + 2\rho(\alpha,\omega)\right]\right\}\Psi_C(\alpha) = 0, \tag{3.17}$$

where we used the relation (2.12) to express it in terms of the density of the scalar field given as [25]

$$\rho(\alpha,\omega) = \rho_o \mathrm{e}^{-3\alpha(1+\omega)}, \tag{3.18}$$

where ρ_o is the density at a certain time and ω is the barotropic parameter assuming that the equation of state of the scalar field is $p = \omega\rho$. The label C of the wave function in Eq. (3.17) stands for *classical*. The first important difference between Eq. (2.53) and (3.17) is that the solutions depend on a different number of variables, although the impact on the EE is not very revealing yet. Presume that the outcomes will be different is misleading ourselves.

It is important to notice that the normalization of the solutions from Eq. (3.17) cannot be obtained from (2.63) since the variable ϕ has been moved into a classical degree of freedom. Nevertheless, the normalization is irrelevant for qualitative results. All divergences will not be regularized, so they are going to be there no matter the normalization we use, and all finite points are going to remain finite. For us, it suffices our requirements in order to study, at least, the qualitative behaviour of the EE.

I analyze in the following some cases centering my attention to the implications of those two different treatments on the EE of a pair universes as I have been doing until this point. That is, the questions to solve are: how the different cases have an impact on the EE? Are they equivalent?

3.6.1 Stiff Matter Dominated Universes

The first case is the trivial one: a pair of flat universes which are filled just with stiff matter [122, 123]. The equation of state for this peculiar fluid reduces to $p = \rho$. The energy of the scalar field is just kinetic, it is, $V(\phi) = 0$.

Attending to the formalism with quantized scalar field, the mass (2.54) vanishes, and the WDW equation (2.53) is simplified as

$$\left[\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2}\right]\Psi_Q(\alpha,\phi) = 0. \tag{3.19}$$

For the alternative WDW equation (3.17), I get

$$\left[\frac{\partial^2}{\partial\alpha^2} + 2\rho_o\right]\Psi_C(\alpha) = 0. \tag{3.20}$$

The solutions to Eq. (3.19) are

$$\Psi_Q(\alpha,\phi) \propto \mathrm{e}^{\mathrm{i}\sqrt{E_\phi}(\alpha\pm\phi)}, \tag{3.21}$$

which is a two-dimensional plane wave, and the solutions to Eq. (3.20) are

$$\Psi_C(\alpha,\phi) \propto \mathrm{e}^{\pm\mathrm{i}\sqrt{2\rho_o}\alpha}, \tag{3.22}$$

which is another plane wave but one-dimensional. These results could have been expected after a careful glance at Eqs. (3.19) and (3.20) with the relation (2.59), since they are differential equations equivalent to the time-independent harmonic oscillator. Therefore, the vacuum state in which we describe the pair of universes will be the same at any point of their evolution, hence the EE is everywhere vanishing.

If one tries to get an everywhere vanishing EE with the algorithm in Section 3.3, the right normalization (2.63) must be taken, otherwise the EE is constant everywhere. This shows up that the right normalization is necessary to get correct results in our calculations.

The pair of stiff matter dominated universes appears to be the trivial case where the EE vanishes. The quantum correlations between the pair are always inexistent. That is why it looks the most simple system to work with in case one wants to avoid entanglement in the multiverse.

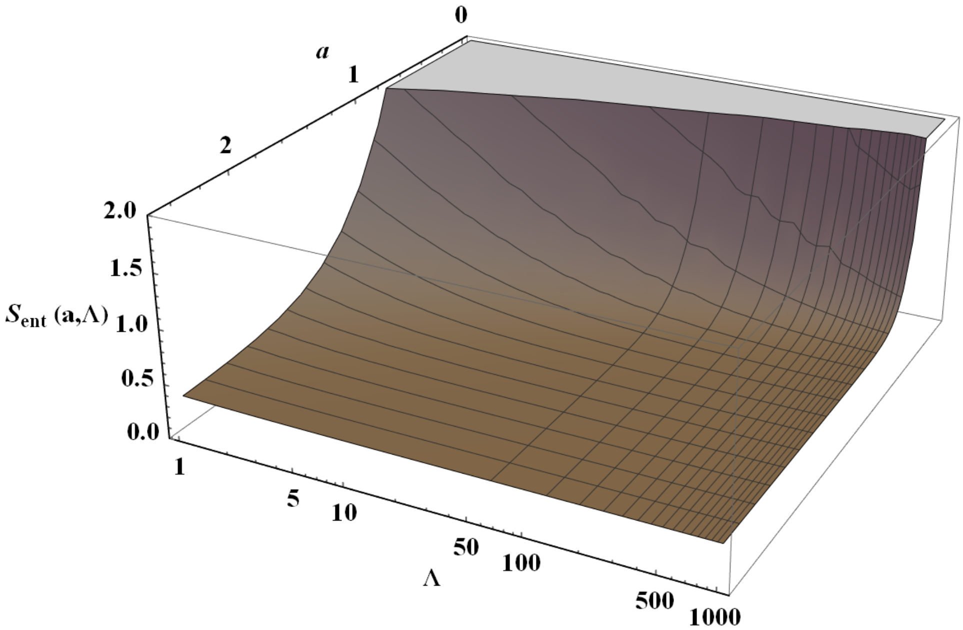

Figure 3.2: Qualitative form of the entanglement entropy of a pair of de Sitter universes. The EE is decreasing as the universe grows, and it decreases faster as the cosmological constant increases.

3.6.2 de Sitter Spacetimes

The second most interesting case I find is the pair of de Sitter universes. Those are flat universes with a positive cosmological constant Λ [25], or a scalar field which follows the equation of state $p = -\rho$. The energy is just potential, it is $V(\phi) \neq 0$, and $\dot{\phi} = 0$.

Whether the repulsive dynamics of the de Sitter universe is given by a cosmological constant or a scalar field is not generally relevant for cosmological purposes. However, here one needs to point out that the uncertainty principle plays an important role in de Sitter universes when it is a quantum scalar field whose kinetic energy has been fixed as $\dot{\phi} = 0$, the one contributing to the matter content. I will elude the debate and the implications of it, and consider that the nature of the cosmological constant is a scalar field whose potential energy is constant, as we described.

The WDW equations (2.53) and (3.17) become identical:

$$\left[\frac{\partial^2}{\partial\alpha^2} + \mathrm{e}^{6\alpha}\frac{\Lambda}{3}\right]\Psi(\alpha) = 0, \tag{3.23}$$

where $\Lambda = 6\rho$. The solutions are

$$\Psi_1(\alpha) \propto \mathcal{J}_0\left[\frac{1}{3}\sqrt{\frac{\Lambda}{3}}\mathrm{e}^{3\alpha}\right], \qquad \Psi_2(\alpha) \propto \mathcal{Y}_0\left[\frac{1}{3}\sqrt{\frac{\Lambda}{3}}\mathrm{e}^{3\alpha}\right]. \tag{3.24}$$

where $\mathcal{J}_\nu(z)$ and $\mathcal{Y}_\nu(z)$ are Bessel functions of the first and second kind, respectively. Both functions are real, so I can use them as inputs for the parameter R in Eq. (2.72). The normalization of those wave functions can be ignored to get a qualitative result of the EE. Thus, I get the EE (2.81) in terms of the scale factor and the cosmological constant, which is shown in Figure 3.2. There, we find that the EE is a monotonically decreasing function after the initial singularity. The cosmological constant looks like if it is the control parameter of the decoherence since the EE gets diluted as fast as the cosmological constant grows.

The pair of de Sitter universes came out to be the one whose EE is identical independently of the treatment of the scalar field, but it is not as trivial as the stiff matter dominated universes of Section (3.6.1).

3.6.3 A General Case

After the two most simple and important cases, a general one is studied where the difference is perfectly seen. For our purpose, I do not need to find the EE everywhere, but to check that there are differences on the treatment. In order to do so, I analyse the behaviour of the EE very close to the initial singularity.

The case that matters is a pair of closed universes with some potential $V(\phi)$. For such case, there is not, in general, analytical solution to the WDW equation (2.53), or a perfect analogy between the classical and the quantum systems, but it is possible to find numerical solutions and study the asymptotic behaviour at early times. The WDW equations (2.53) and (3.17) are for the model:

$$\left[\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} - \mathrm{e}^{4\alpha} + 2V(\phi)\mathrm{e}^{6\alpha}\right]\Psi_Q(\alpha) = 0, \tag{3.25}$$

and

$$\left[\frac{\partial^2}{\partial\alpha^2} - \mathrm{e}^{4\alpha} + 2\rho_o\mathrm{e}^{3\alpha(1-\omega)}\right]\Psi_C(\alpha) = 0, \tag{3.26}$$

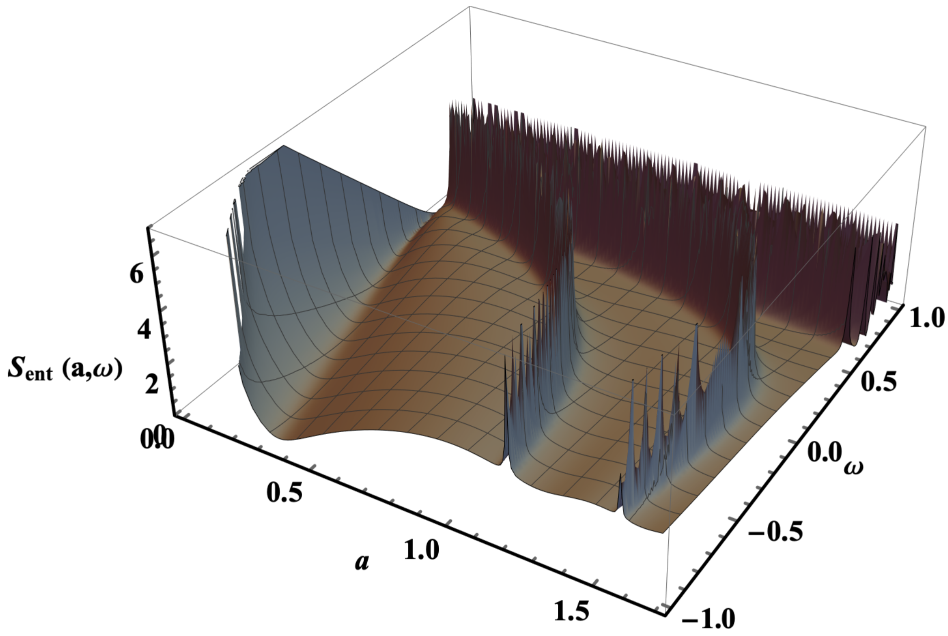

Figure 3.3: Entanglement entropy of a pair of universes in a general case given by WDW equation (3.26) with $\rho_o = 1/2$. The series of peak chains of the image are caused of the numerical method and the rapidly varying wave functions, so the image must be considered smooth everywhere. Thus, the EE is divergent at the initial singularity and decreases as the universe expands, the faster the decrease the bigger is the parameter ω.

respectively. Here I used the relation (3.18). We can be safe studying the classical case with the barotropic parameter ω into the interval $[-1, 1]$ since

$$\omega = \frac{p}{\rho} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}, \tag{3.27}$$

and we assume that the kinetic energy of the field is always positive. This is rather general even if we do not know the form of $V(\phi)$.

First, let me start choosing $\rho_o = 1/2$ and finding the numerical solutions to the WDW equation (3.26) for different values of $\omega \in [-1, 1]$ fulfilling the boundary condition (2.63). The EE is calculated taking the real and the imaginary part of the first solution we obtain. We depicted it in Figure 3.3. The numerical method includes derivatives of the wave function, that at some points is rapidly varying. It induces some numerical imperfections on the surface and must be taken away. Apart from that, the EE is divergent at the initial singularity and decreases as the universe expands. Again, the parameter ω

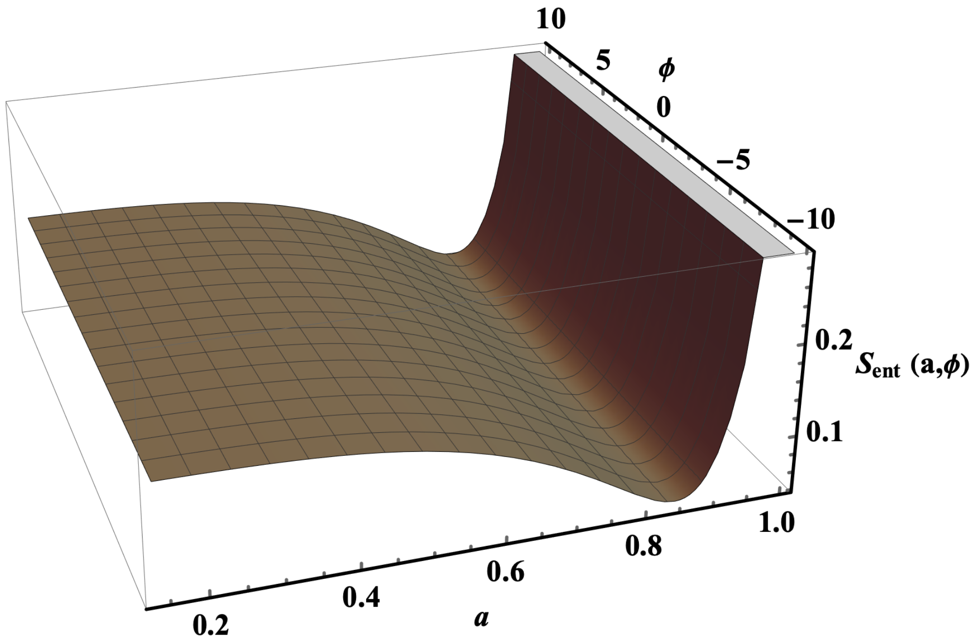

Figure 3.4: Entanglement entropy of a pair of universes in a general case given by WDW equation (3.25). This plot is equivalent to the one I should have obtained just in the region very close to the initial singularity, where the EE is finite everywhere. Exactly, this plot corresponds to a closed universe with a massless scalar field whose maximum is at $a = 1$, almost identical to the one found in Section 3.5.

seems to control how fast the EE decreases, being steeper as the parameter ω grows.

In the case of the quantized scalar field, the WDW equation (3.25) cannot be solved unless the shape of the potential $V(\phi)$ is known. However, at the very early times, the last term into the brackets can be neglected. After the approximation, I obtain the solutions

$$\Psi(\alpha,\phi) \propto \mathcal{I}_{\pm \mathrm{i}\frac{\sqrt{E_\phi}}{2}}\left[\frac{1}{2}\mathrm{e}^{2\alpha}\right]\mathrm{e}^{\pm \mathrm{i}\sqrt{E_\phi}\phi}, \tag{3.28}$$

applying the separation as in Eqs. (2.59). Ignoring the normalization, the EE is calculated and shown in Figure 3.4. This plot is therefore just valid for comparison with Figure 3.3 at very early times, since it corresponds to the case studied previously in Section 3.5, but the distribution function has been taken as[4] $A(k) = \delta(k - E_\phi)$, for simplicity. The approximation is enough to prove what I wanted, since the EE is finite at those

[4]This distribution, for it to be symmetric around $k = 0$, is $A(k) = \delta(k - E_\phi) + \delta(k + E_\phi)$, but since I integrate from 0 to ∞, the result is the same.

early times. In comparison to what it was found for the classical treatment, that was a divergent behaviour, one can check that it makes the difference, at least close to the initial singularity.

The conclusion is that there is a difference touching the treatment of the scalar field. Its quantization works like a regulator of the quantum correlations at the beginning of the universe, hence the EE gets finite values, while for the classical treatment we find a divergent EE. Besides, there are singular models, as we studied in Sections 3.6.1 and 3.6.2 for which the EE is independent of how we treat the scalar field.

3.7 Oscillating Universes

In light of the results from previous sections, the EE is divergent at the maximum of the expansion of the universe. To know whether the same happens at any other critical points is the aim of this section. I study the EE at any kind of critical points, not just at maxima as seen in Section 3.5, but using a classical treatment of the scalar field. I consider the case of a pair of universes whose dynamics is described by a perfect sine, where there are maxima, minima, and inflection points. The content of the universe was analyzed in Ref. [124], and similarly to this one, it has been studied in many different studies [125–131].

In the aforementioned model, the universes contain wall-like matter, string-like matter, and a negative cosmological constant (for details about the non-standard matter go through Ref. [132]). The specific evolution of the universe I consider is

$$a(t) = -\frac{3}{2\Lambda}\left[A\sin\left(\sqrt{-\frac{\Lambda}{3}}t\right) + C_w\right], \tag{3.29}$$

where

$$C_w > A = \sqrt{C_w^2 + \frac{4}{3}\Lambda k'}, \tag{3.30}$$

is the constant density parameter due to wall-like matter, we define $k' = \mathtt{k} - C_s$, and C_s is the constant density parameter due to string-like matter. The minima and the maxima of

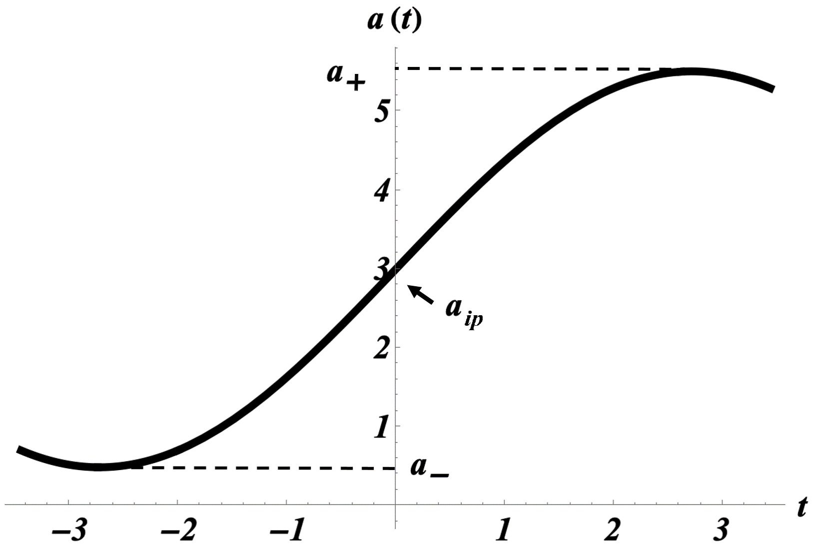

Figure 3.5: Classical evolution (3.30) of an oscillating closed universe with $\Lambda = -1$, $C_s = 0.1$, and $C_w = 2$. Here, I highlight the maximum, the minimum, and the inflection point as a_+, a_-, and $a_{\rm ip}$, respectively.

the evolution are located at

$$a_{\pm} = -\frac{3}{2\Lambda}(\pm A + C_w). \tag{3.31}$$

The Friedmann equation, whose solution is (3.30) as desired, is

$$H^2 = \frac{\Lambda}{3} + \frac{C_w}{a} - \frac{k'}{a^2}, \tag{3.32}$$

that comes from the Lagrangian

$$L = \frac{1}{2}\int \mathrm{d}N a^3 \left[\frac{H^2}{N^2} - \frac{\Lambda}{3} - \frac{C_w}{a} + \frac{k'}{a^2}\right], \tag{3.33}$$

where N is the lapse function, selected to be $N = 1$ in order to recover (3.32). The corresponding Hamiltonian to this Lagrangian is

$$H = -\frac{p_a^2}{a} - k'a + C_w a^2 + \frac{\Lambda}{3}a^3. \tag{3.34}$$

Once I get the Hamiltonian, the quantization of the momentum p_a yields, following the same steps as in Section 2.2.2, the WDW equation

$$\left[\frac{\partial^2}{\partial\alpha^2} - k'\mathrm{e}^{4\alpha} + C_w \mathrm{e}^{5\alpha} + \frac{\Lambda}{3}\mathrm{e}^{6\alpha}\right]\Psi(\alpha) = 0. \tag{3.35}$$

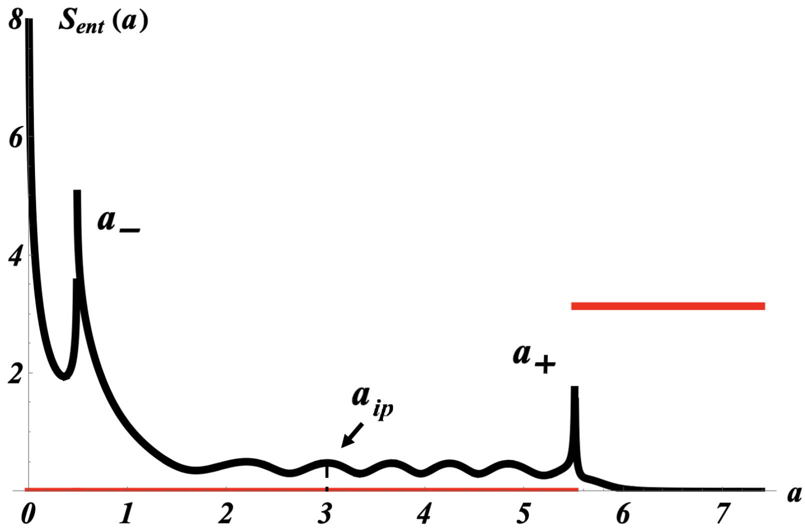

Figure 3.6: Entanglement entropy of an oscillating closed universe with $\Lambda = -1$, $C_s = 0.1$, and $C_w = 2$. Here I highlight the maximum, the minimum and the inflection point as a_+, a_- and $a_{\rm ip}$, respectively. The black line is the real part of the EE, and the red one is its imaginary part.

Here one notices that the scalar field has been considered classically and the densities of each kind of matter has been included into the density term in Eq. (3.17).

In order to find the EE, we need two real solutions of Eq. (3.35). It is not possible to do it analytically, so I find numerical solutions imposing as boundary condition the solution in the asymptotic region $\alpha \to -\infty$:

$$\Psi^{(1)}(\alpha) \propto \mathcal{I}_0\left[\frac{\sqrt{k'}}{2}\mathrm{e}^{2\alpha}\right], \qquad \Psi^{(2)}(\alpha) \propto \mathcal{K}_0\left[\frac{\sqrt{k'}}{2}\mathrm{e}^{2\alpha}\right], \tag{3.36}$$

where the two last terms into the brackets in Eq. (3.35) are suppressed. Here $\mathcal{I}_\nu(z)$ and $\mathcal{K}_\nu(z)$ are modified Bessel functions of the first and the second kind, respectively. With the values $\Lambda = -1$, $C_s = 0.1$, and $C_w = 2$, a closed universe evolves like (3.29), which is shown in Figure 3.5. The inflection point has been labeled as $a_{\rm ip}$. For those parameters, the outcome of the EE is in Figure 3.6. It is the first time we explicitly draw the imaginary part of the EE, which is the red line. The black line is the real part. As commented in Section 3.4, the EE is real in the classically allowed region, which is between a_- and a_+, but also in the region from $a = 0$ to a_- [124]. The latter case is not useful for us as there is not a minimum at $a \neq 0$ as it is in a_-. From a_+ on, it is not accessible classically, so

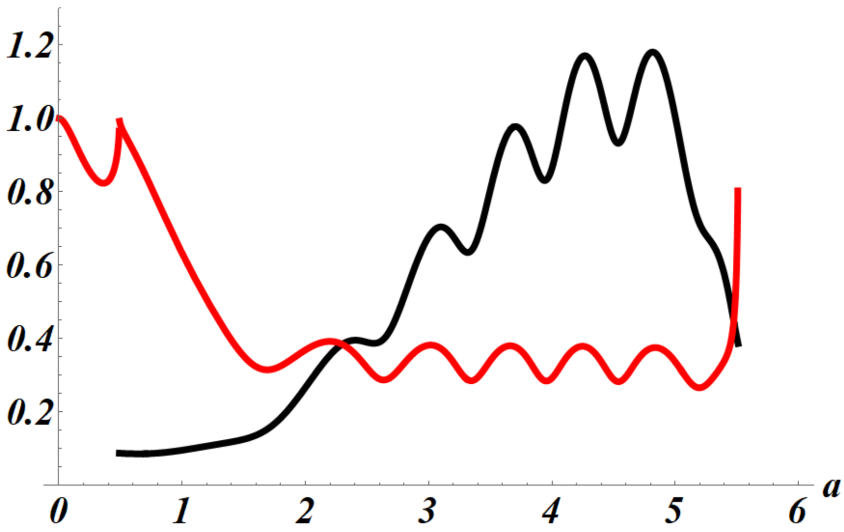

Figure 3.7: Temperature of entanglement, in black, and the parameter Q, in red, of an oscillating closed universe with $\Lambda = -1$, $C_s = 0.1$, and $C_w = 2$. The temperature has been multiplied by 1/10 to fit it into frame.

the EE contains an imaginary part with the value of π.

Besides, the EE is divergent at the critical points a_- and a_+, but it is still finite at the inflection point. Also, at $a = 0$, one confirms that since the fields are not quantized, the EE diverges as inferred in Section 3.6.3. Then, apart from the initial singularity, the EE diverges where the Hubble parameter $H = \dot{a}/a$ vanishes.

Finally, taking advantage of this useful and simple model, I calculate the temperature of entanglement (2.79) and the parameter Q (2.80). The results are in Figure 3.7. The parameter Q has a shape which coincides with the EE, being equal to one at the divergences of the EE, while the temperature is still finite everywhere. Considering the von Neumann entropy as the right measurement of the quantumness of a system, one does not get a good impression from the temperature to be a valuable quantity to measure the quantumness as it was suggested in Ref. [111].

Type	Name	t	$a(t_s)$	$\varrho(t_s)$	$p(t_s)$	$\dot{p}(t_s)$	$w(t_s)$	T	K
0	big bang (BB)	0	0	∞	∞	∞	finite	strong	strong
I	big rip (BR)	t_s	∞	∞	∞	∞	finite	strong	strong
I_l	little rip (LR)	∞	∞	∞	∞	∞	finite	strong	strong
II	sudden future (SFS)	t_s	a_s	ϱ_s	∞	∞	finite	weak	weak
II_a	big brake (BBr)	t_s	a_s	0	∞	∞	finite	weak	weak
III	finite scale factor (FSF)	t_s	a_s	∞	∞	∞	finite	weak	strong
III_a	big freeze (BF)	t_s	0	∞	∞	∞	finite	weak	strong
IV	big separation (BS)	t_s	a_s	0	0	∞	∞	weak	weak
V	w-singularity (w)	t_s	a_s	0	0	0	∞	weak	weak

Table 3.1: Classification of exotic singularities using the definitions of Tipler and Królak.

3.8 Exotic Singularities

Once I have studied what happen with the EE of a pair of universes at their origin, at critical points and inflection points, one could wonder: how does it behave at exotic singularities? Those kind of singularities are nonstandard [133–136] ones where the scalar field gets different values from the standard singularity in General Relativity, where pressure and density blow up.

A summarize of those exotic singularities is done in Table 3.1 [134, 137]. There we explain the name and its classification in types, the time t at which each singularities occurs and the value of the scale factor a at that time t. Also, the density, the pressure, the slope of the pressure and the quotient $w = p/\rho$. Finally, we present the classification of these singularities using the definitions of Tipler [138] and Królak [139], where the singularities are called *strong* if, in general, some components of the tensors

$$^{(\mathrm{T})}I^i_j(\tau) = \int_0^\tau \mathrm{d}\tau' \int_0^{\tau'} \mathrm{d}\tau'' \left| R^i{}_{ajb} u^a u^b \right|, \qquad ^{(\mathrm{K})}I^i_j(\tau) = \int_0^\tau \mathrm{d}\tau' \left| R^i{}_{ajb} u^a u^b \right|, \tag{3.37}$$

diverge, respectively, at $\tau = \tau_s$, where τ is the proper time and τ_s is the moment where the singularity appears, or weak, in general, if all the components are finite at any time.

Among them, I will focus on four different types. I have already analyzed the type 0 singularity which is the Big Bang singularity, so the first one is the type II_a singularity or Big Brake [140–142], where the density vanishes and the pressure diverges. The second one will be the type III_a singularity or Big Freeze [143, 144], where pressure and density diverge, but at a finite time τ_s. The third type will be the type IV singularity or Big

Separation [145], for which the pressure and density vanish at some $\tau = \tau_s$. And the last one which is a type I_l singularity called Little Rip [146], where pressure and density diverge asymptotically, after an infinite period of time in the future.

3.8.1 Type II_a Singularity – Big Brake

The type II_a singularity called Big Brake belongs to the class of Sudden Future Singularity and it can be reproduced in a flat FLRW universe filled with a scalar field whose equation of state goes like

$$p = -\frac{A}{\rho^\beta}, \tag{3.38}$$

which is the one of a generalized Chaplygin gas [147], where $A < 0$ is a constant, and as an example, we take $\beta = 1$.

The expression of the density in terms of the scale factor is

$$\rho = \sqrt{\frac{B}{a^6} - A}, \tag{3.39}$$

where $B > 0$. That way, the density vanishes at $a_s = a(t_s) = (B/A)^{1/6}$, where t_s is the time at which the singularity is found. The classical evolution of a universe like this starts at $a = 0$, then expands, and has a turning point at a_s. Thus,

$$\dot{a}(t_s) = 0, \qquad \ddot{a}(t_s) \to -\infty, \qquad \frac{A}{p(t_s)} = \rho(t_s) \to 0. \tag{3.40}$$

The WDW equation (3.17) can be adapted to this case replacing the density (3.40) and setting $\Lambda = 0$ and $\mathtt{k} = 0$. The boundary condition I use is again to fix the solution to the asymptotic one obtained at $\alpha \to -\infty$, where $\rho \sim \sqrt{B}\mathrm{e}^{-3\alpha}$, which are

$$\Psi^{(1)}(\alpha) = \mathcal{J}_0\left[\frac{\sqrt{8}B^{1/4}}{3}\mathrm{e}^{3t/2}\right], \qquad \Psi^{(1)}(\alpha) = \mathcal{Y}_0\left[\frac{\sqrt{8}B^{1/4}}{3}\mathrm{e}^{3t/2}\right], \tag{3.41}$$

where $\mathcal{J}_\nu(z)$ and $\mathcal{Y}_\nu(z)$ are Bessel functions of the first and second kind, respectively. Using those functions to obtain the global solutions to (3.17), the EE was calculated, with $B = -A = 1$, and it is shown in Figure 3.8. There, the real part of the EE is drawn in black, and the imaginary part in red. Besides, I added the temperature in blue,

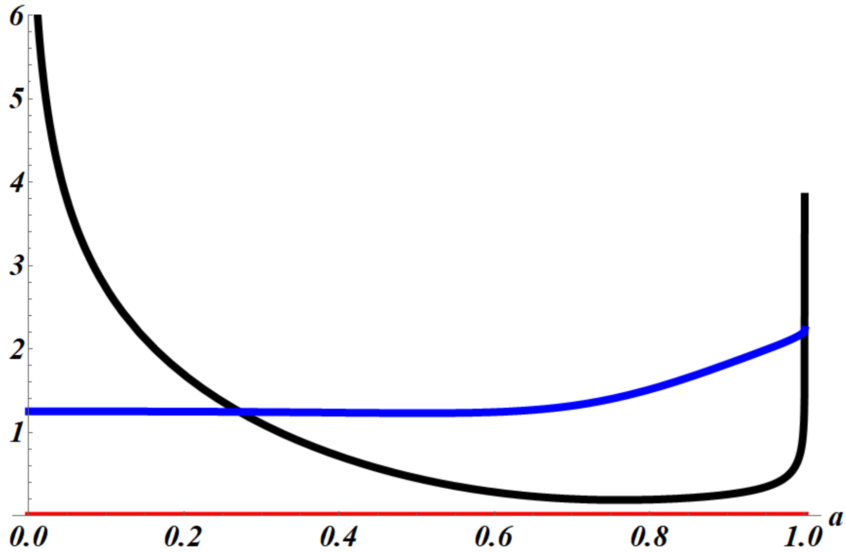

Figure 3.8: The real part of the entanglement entropy, in black, its imaginary part, in red, and the temperature of entanglement, in blue, of a flat universe with a scalar field whose equation of state is (3.40). Here: $\beta = 1$, $B = -A = 1$. The EE diverges at the initial singularity and at the Big Brake singularity located at $a_s = 1$. The temperature has been multiplied by 5 to fit in the frame.

multiplied by a factor of 5, for comparison. As expected, the EE diverges at $a = 0$ since the scalar field is still classical, and since $\dot{a}(t_s) = 0$ at the Big Brake singularity, i. e., at $a_s = 1$, the EE is divergent. The temperature does not reproduce any of the expected quantum properties since it is kept finite everywhere and any point along the evolution of the universe, not even the initial singularity, gains special importance. The conclusion is the same as in Section 3.7: the temperature seems not to be a good measurement of the quantumness of a bipartite system like ours, considering the von Neumann entropy to be the correct way. It is important to point out that here the EE may be divergent due to the fact that the Hubble parameter vanishes and not because of the very singularity by its own. What we can say is that it is not regularized because of this special kind of singularity.

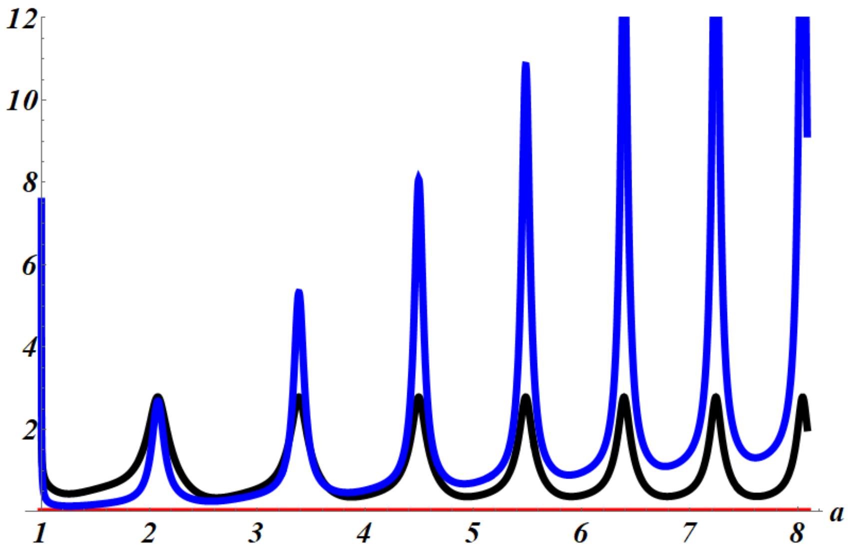

Figure 3.9: The real part of the entanglement entropy, in black, its imaginary part, in red, and the temperature multiplied by 1/10, in blue, of a flat universe with a scalar field whose equation of state is (3.40), with $\beta = -2$, $B = -A = 1$. The Big Freeze singularity appears at $a_s = 1$, where the EE and the temperature diverge. Both are finite everywhere, but the temperature is oscillating and growing as the universe expands.

3.8.2 Type III$_a$ Singularity – Big Freeze

The type III$_a$ singularity called Big Freeze belongs to the class of Finite Scale Factor Singularity and it can be given in a flat universe with a scalar field whose equation of state is (3.40), and $A < 0$ and $\beta < -1$ are constant. However, in this case, the singularity is a minimum of expansion.

The density in terms of the scale factor is

$$\rho = |A|^{1/(1+\beta)} \left[\left(\frac{a_s}{a} \right)^{3(1+\beta)} - 1 \right]^{1/(1+\beta)}, \tag{3.42}$$

where

$$a_s = a(t_s) = \left| \frac{B}{A} \right|^{1/3(1+\beta)}, \tag{3.43}$$

is the minimum size of the universe and B is a constant of integration. The Big Freeze singularity at a_s has the properties

$$\rho(t_s), p(t_s) \to \infty, \qquad \dot{a}(t_s) \to 0. \tag{3.44}$$

Using the WDW equation (3.17) with the same boundary condition procedure as in the previous Section (3.8.1), I get the EE which is shown, together with the temperature of entanglement, in Figure 3.9. I used the values $B = -A = 1$ and $\beta = -2$, for which the Big Freeze singularity is located at $a_s = 1$. There, the real part of the EE is drawn in black, its imaginary part in red, and the temperature in blue. As for this model the universe expands forever, there is no maximum, so the only interesting point is the singularity at $a_s = 1$, for which the EE and the temperature diverges. For the latter times, the EE remains finite and oscillating, while the temperature oscillates with peaks which are increasing as the universe expands. The temperature is again behaving against the logic, since the entanglement disappears as the universe increases. Again, the divergent behaviour cannot be explained uniquely because of the exotic singularity, but because the Hubble parameter vanishes, so the only thing we can say is that it is not finite because of the new properties of the singular point.

3.8.3 Type IV Singularity – Big Separation

A type IV singularity named Big Separation is found in a universe with a scalar field whose equation of state is (3.40) and whose density is given by (3.42) with $B > 0$ and $\beta \in [-1/2, 0)$. The singularity is then a maximum of expansion at a_s as in (3.43). There

$$p(t_s), \rho(t_s) = 0, \qquad \dot{a}(t_s) = 0. \tag{3.45}$$

Following the same steps as before, the EE and the temperature of entanglement are calculated and depicted in Figure 3.10. We do not find too many differences between those functions and the ones in Figure 3.8 for a Big Brake. At the initial singularity and at the Big Separation, the EE diverges while the temperature is kept finite everywhere, showing once more that it seems to give no information about the quantum state of the universe. Once more, as in the previous cases, we cannot state anything about the nature of the divergent behaviour of the EE due to the exotic singularity, but due to the Hubble parameter.

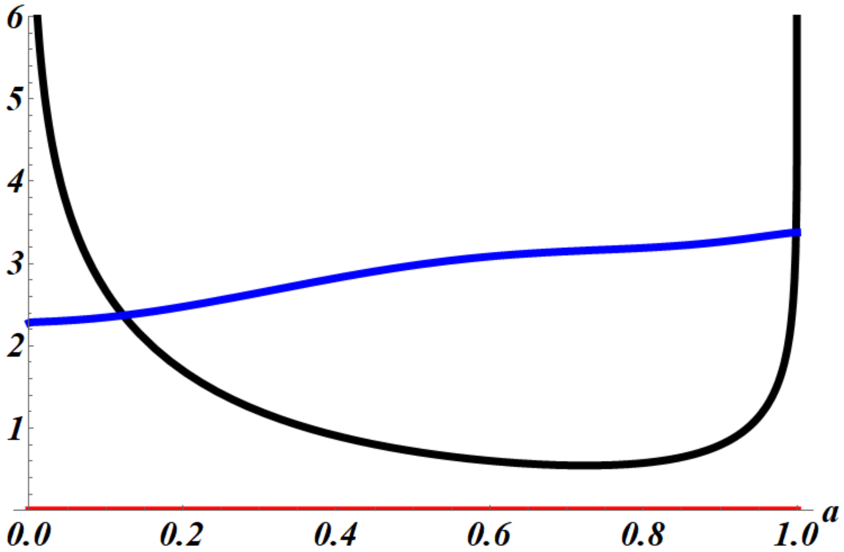

Figure 3.10: In black, the real part of the entanglement entropy, in red, its imaginary part, and in blue, the temperature of entanglement multiplied by 10. The EE diverges at $a = 0$ and at the Big Separation singularity at $a_s = 1$. The temperature is finite everywhere. Here, $\beta = -1/2$, and $B = -A = 1$.

3.8.4 Type I$_l$ Singularity – Little Rip

A flat universe with a Little Rip singularity appears when a scalar field with equation of state

$$p = -\rho - A\sqrt{\rho} < 0, \tag{3.46}$$

fills it. Here, $A > 0$ is a constant. The density is hence following the relation

$$\rho = \rho_o \left[\frac{3A}{2\sqrt{\rho_o}} \ln\left(\frac{a}{a_o}\right) + 1 \right]^2, \tag{3.47}$$

in terms of the scale factor, where ρ_o and a_o are constants of integration accounting for the value of the density and the scale factor at a certain time, respectively.

The point under study is

$$a_s = a_o \mathrm{e}^{-\frac{2\sqrt{\rho_o}}{3A}}, \tag{3.48}$$

where the density vanishes. Introducing (3.47) into (3.17), one finds that the frequency $\omega^2(\alpha)$ in the WDW equation is proportional to ρ, therefore the EE is also expected to diverge there. The boundary condition is obtained as we proceeded in the last sections, and the outcome is shown in Figure 3.11 together with the temperature of entanglement and the parameter Q, which is introduced here for completion.

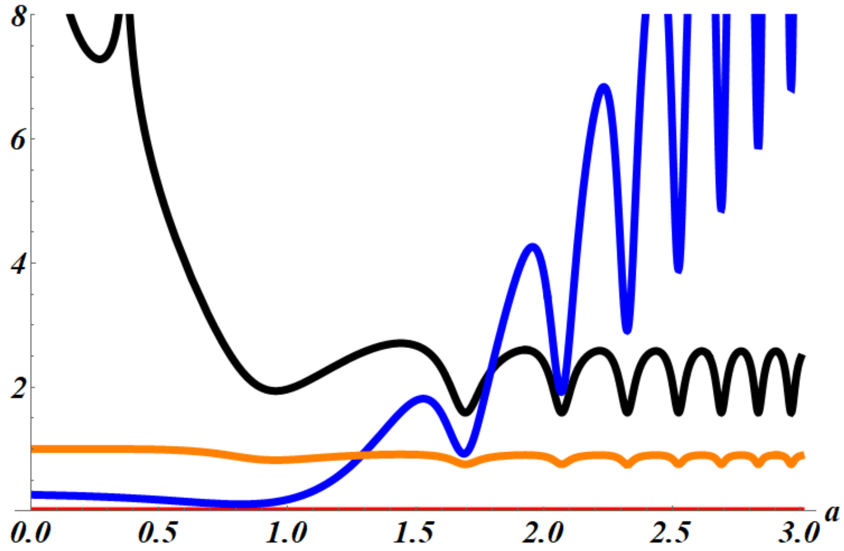

Figure 3.11: The real part of the entanglement entropy in black, its imaginary part in red, the temperature of entanglement multiplied by 1/20 in blue, and the parameter Q in orange. The EE diverges at the initial singularity, and at the Little Rip singularity located at $a_s = \mathrm{e}^{-1}$. Elsewhere, it remains finite. The parameter Q behaves everywhere as the EE, but it is equal to one when the EE diverges. The temperature is finite but increasing, while it oscillates as the universe gets bigger.

The EE diverges at $a = 0$ and at the Little Rip singularity a_s as expected, and it remains finite at any other point. The parameter Q is also following the shape of the EE, being equal to one when the EE diverges. The temperature, however, is equal to one last time diverging for late times but finite at any finite time, not following a proper behaviour. Let me remark that this model is the first one which serves as an example where the EE diverges for noncritical points.

3.9 Relation of the Entanglement with the Hubble Parameter

After the analysis of all those models in previous sections, we concluded that the dependence of the EE on the Hubble parameter is the important EE characteristic, since the EE is divergent everywhere when $H = \dot{a}/a = 0$. It is then interesting to find its relation to the critical points of the classical evolution of the universe.

For such a task, I consider the case of a closed universe filled with a scalar field whose spectrum is $A(k) = \delta(k-1)$. The WDW equation (2.62) is then

$$\left[\frac{\partial^2}{\partial\alpha^2} + 1 - \mathrm{e}^{4\alpha}\right]\Psi(\alpha) = 0, \tag{3.49}$$

whose solutions are

$$\Psi(\alpha) = \mathcal{I}_{\pm\frac{\mathrm{i}}{2}}\left[\frac{\mathrm{e}^{2\alpha}}{2}\right]. \tag{3.50}$$

Taking the real and the imaginary part of any of them as inputs for R (2.72), the EE given by the von Neumann entropy (2.81), and the Tsallis and Rènyi entropies (2.82), are calculated. They are shown in Figure 3.12 from which we can see that the overall behaviours are similar. The von Neumann entropy is drawn in blue, while the Renyi (red) and Tsallis (black) entropies are the solid ones with $q = 1.5$, and the dotted ones with $q = 0.7$.

At $a = 1$, where the model gets a classical maximum of expansion, all entropies diverge. There is no problem for the generalized entropies to diverge to negative values since they are not bounded from below as the von Neumann one. Any kind of divergence denotes a high quantumness at the point. It is also interesting to notice that they are all finite at $a = 0$, where the initial singularity takes place, i. e., when the quantization of the scalar field regularizes also the generalized entropies.

Getting advantage of this very simple model, I would like to get the asymptotic behaviour of the von Neumann entropy around the maximum at $a = 1$. My hope was to

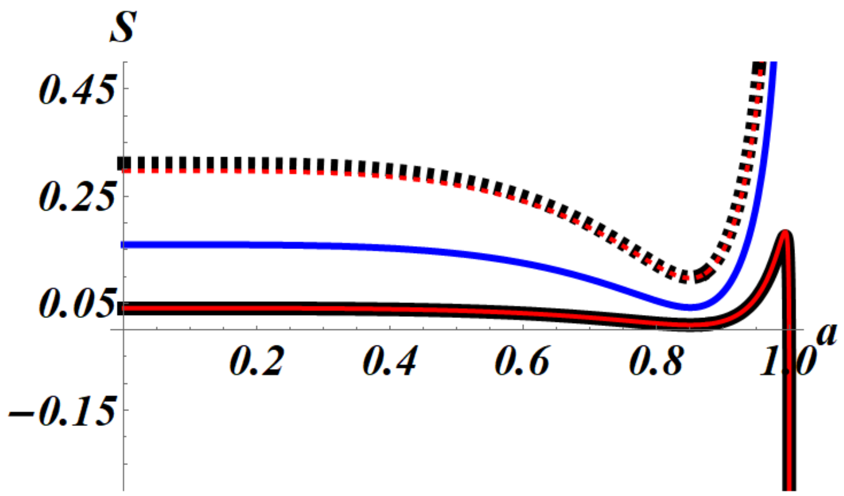

Figure 3.12: Von Neumann, Tsallis and Rènyi entropies in blue, black and red, respectively. The solid lines and dotted lines are for the parameters $q = 1.5$ and $q = 0.7$, respectively, of the Tsallis and Rènyi entropies. A very similar form is obtained for all of them. They are finite at the initial singularity and diverge at the maximum of expansion.

get an asymptotic form like

$$S^{\rm N}(H) \sim \frac{1}{H^2}, \tag{3.51}$$

in order to relate it to the entropy of gravitational horizons, like Hubble or black hole horizons [148]. Unluckily, after fitting some of the points close to the maximum to a function like

$$S^{\rm N}(H) = c_o + \frac{c_1}{H} + \frac{c_2}{H^2} + \frac{c_3}{H^3}, \tag{3.52}$$

where the expected condition was $c_2 \gg c_o, c_1, c_3$, I found the blue line in Figure 3.13 which is not a very good fit since the exact entropy is the black line. It turned out that the von Neumann entropy fits almost perfectly, with goodness coefficient $r^2 \sim 1$, to

$$S^{\rm N}(H) = c_o - c_1 \ln(H), \tag{3.53}$$

with $c_1 \approx 1$. Since c_o is up to the normalization of the wave functions, then one can say that $S^{\rm N} \sim -\ln(H) \sim I(H)$, where $I(H)$ is the Shannon information content [149]. The fit is also in Figure 3.13 in red.

Finally, I also wanted to show if the logarithmic shape also holds for the extended entropies (2.82). The fits to the Tsallis and Rènyi entropies are in Figure 3.14. In it, the

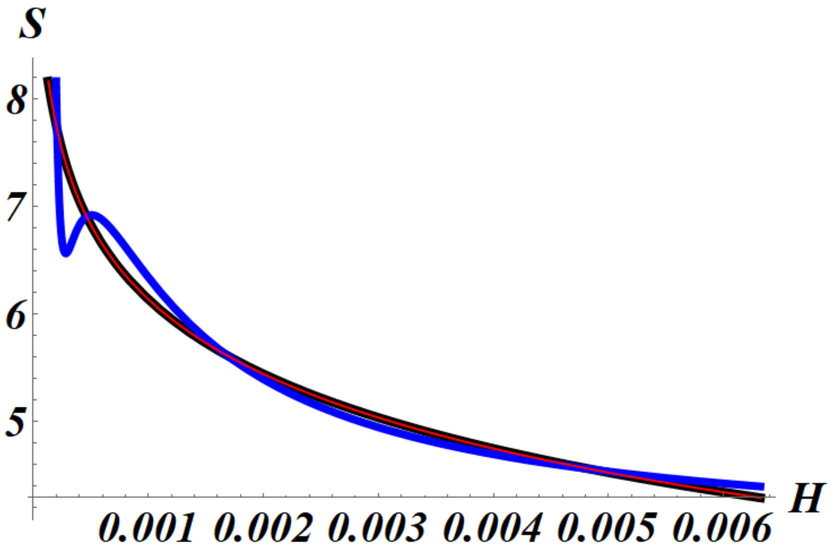

Figure 3.13: The von Neumann entropy (black), and its fits (3.52) and (3.53) in blue and red, respectively.

solid lines show the Rènyi (black) and Tsallis (blue) entropies in terms of H for $q = 0.7$, and their fits in red and green, respectively. The dotted lines show the absolute values of the Rènyi (black) and Tsallis (blue) entropies for $q = 1.5$, and their fits in red and green, respectively. It looks that the Tsallis entropy does not fit as well as the others to the logarithmic shape.

3.10 Decoherence and Our Hypothetical Twin

This chapter up to here has been dedicated to study the theoretical aspects of the EE of a pair of universes. I have gone into different models and some general conclusions have been obtained. It was done for the ideal situation in which no other interaction is possible in the multiverse. A more realistic scenario should not be so naive and it would include interactions with other universes or any other entities. They would play the role of an environment, whose quantum properties decohere and all crossed interactions tend to disappear [116, 150]. This is the customary scenario of a highly interacting theory.

In the hypothetical case in which our universe was born at the same time as a twin, the interaction between them would only affect to the very early times, since the dynamics

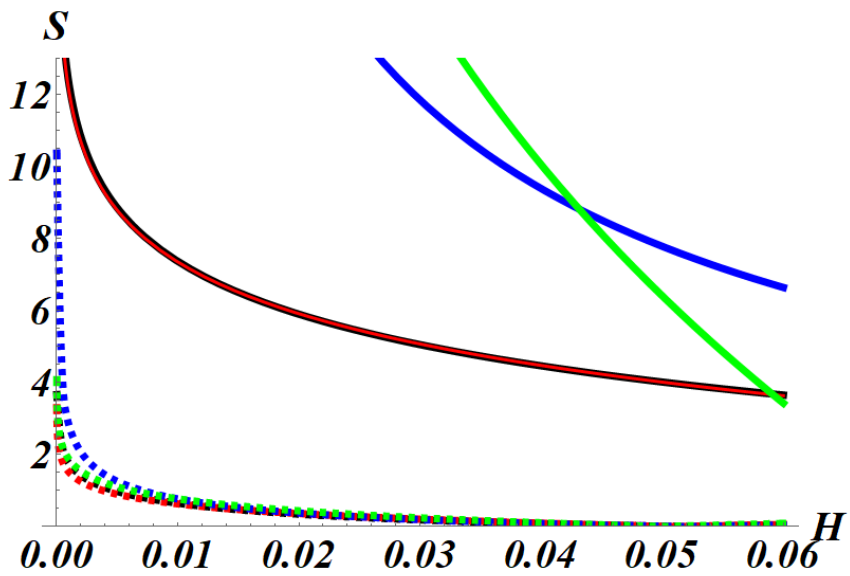

Figure 3.14: Extended entropies and their fits. The solid lines show the Rènyi (black) and Tsallis (blue) entropies in terms of H for $q = 0.7$, and their fits in red and green, respectively. The dotted lines show the absolute values of the Rènyi (black) and Tsallis (blue) entropies for $q = 1.5$, and their fits in red and green, respectively.

of our universe can be (almost) recovered using a classical theory of gravity as General Relativity. Our universe has never been in a critical point [25], and inflection points are not quantumly relevant, which is sad because our universe has just gone through one of those when dark energy started to dominate our present universe, so decoherence is expected to constantly dilute any other clue of a twin universe. The question whether our twin universe exists or not is almost impossible to answer yet. The only window we have to check its existence is the Cosmic Microwave Background (CMB) at its low multipoles, or future experiments able to go deep into the very early stages of our universe, as the spectrum of primordial gravitational waves. Even if the expectation of discovering it is low in the close future, I will dedicate next Chapter to find the imprints on the CMB of the interaction with our lost twin.

4

Falsiying the Multiverse Hypothesis

4.1 Posing the Problem of Falsifiability

A physicist is not a philosopher. A physicist is the one who thinks about Nature and the laws which describe it. A philosopher is allowed to think. About anything, even Nature. When a physicist works, he should not be allowed to think as a philosopher. It is not useful for us to talk about a twin universe if we are not able to detect it. In case it exists, the situation is equivalent to as if it does not exist at all. However, it is a physicist job to check when a theory can be falsifiable, and keep thinking constantly whether a metaphysical theory is really metaphysical or not. Our methodology is based on the approach by Popper [151] that any esoteric theory is falsifiable if it contains a predictive result of an experiment to verify if such theory is correct.

The idea of a twin universe is somewhat abstract and may look irrelevant for a physicist. But we have checked that in the representation in which the number of universes remains constant, which is a very natural one, the EE is, in general, unavoidable, and there must be signs into our universe about it. In this chapter, I use what we have learnt about the quantum relation of the pair in order to impose a heuristic interaction between them, and extract the hints our twin could have left in the spectrum of the CMB [152]. The angular spectrum of the CMB, which is directly measured with high resolution by Planck satellite [12, 153], is what I consider here to be the first good indicator of a theory which modifies the earliest period of the universe. Also, the reason to use a heuristic interaction is that we have no clue about the way they are intermingled.

From the Chapter 3, we concluded that the EE is directly related to the Hubble parameter, such that

$$\lim_{H\to 0}\left[S^{\rm N}(a,H)\right]\to\infty. \tag{4.1}$$

It is not intrinsic from the diagonal part of a Hamiltonian (2.68) to see any entanglement but from the interaction terms, therefore, a system which fulfills the condition (4.1), is also subjected to the condition

$$\lim_{H\to 0}\left[H_{\rm int}(a,H)\right]\to\infty. \tag{4.2}$$

Besides, we can impose a new condition related to the formation of universes [154] via the decay of a false vacuum state. The formation of bubbles is exemplified through the Coleman-DeLuccia potential [155], which is an almost-symmetric quartic potential like a slightly damaged Mexican hat, containing two minima with different altitudes in which quantum tunneling is allowed. One of those minima is the true vacuum, for which the energy is a global minimum, and the other one is the false vacuum, with the possibility for transition to the real vacuum state. Such a jump, which occurs locally in the *space* we settle everything, is effectively the creation of bubbles of true vacuum in a sea of false vacuum.

Let us consider the simplest interaction between a set of universes $\{\Psi_i\}$, which depend only on the scale factor, as the one-to-one interaction

$$H_{\rm int}=\sum_{n=1}^{N}a\lambda^2(a)\left(\Psi_{n+1}-\Psi_n\right)^*\left(\Psi_{n+1}-\Psi_n\right), \tag{4.3}$$

similar to a nearest neighbour interaction, with periodic boundary conditions $\Psi_0=\Psi_{N+1}$, where N is the number of universes, and $\lambda^2(a)$ is an unknown coupling function. It was found for this case [154] that the probability of the false vacuum to decay goes like

$$\Gamma \propto \mathrm{e}^{-B}, \qquad \text{with} \qquad B \propto \left[\frac{a^4}{\lambda^2(a)}\right]^3. \tag{4.4}$$

It is then necessary that

$$\lim_{a\to\infty} [\Gamma] \to 0, \tag{4.5}$$

which implies that $\lambda^2(a)$ is a function which is smaller than a^4 for large enough values of a. The effects on the CMB of a large amount of universes in the multiverse interacting as in Eq. (4.3) has been calculated [20], but the results are not useful for us since our interaction is only between a pair of universes and the coupling function $\lambda^2(a)$ is also dependent on the Hubble parameter as I demanded previously. However, the condition (4.5) holds after one changes $\lambda^2(a)$ to $\lambda^2(a, H)$ in Eqs. (4.4).

A good question with a hard answer arises: what is the shape of the coupling function $\lambda^2(a, H)$? In principle, it is impossible to know. The only thing we know is that it should fulfill the conditions (4.1) and (4.5). In the next section, I will introduce an example for it.

4.2 Modeling the Pair Interaction

The mathematical setup is following an outline whose aim is to obtain the equation of motion of the universe in time, it is the modified Friedmann equation, containing an extra term due to the interaction of the pair. In order to get to that point, I write the Hamiltonian of each universe, and then build the total Hamiltonian of both universes by adding the term of interaction.

Let us start then by writing the Lagrangian of a single flat FLRW universe with a scalar field and its mode perturbations $\{v_k\}$ in term of the conformal time $\mathrm{d}\eta = \mathrm{d}t/a(t)$ [20, 156, 157]:

$$L = \frac{1}{2}\left[-(a')^2 + a^2(\phi')^2 - 2a^4V(\phi) + \sum_k (v_k' v_k^{*\prime} + \omega_k^2 v_k v_k^*)\right], \tag{4.6}$$

where the prime denotes the derivative with respect to the conformal time, and the frequency

$$\omega_k^2 = k^2 - \frac{z''}{z}, \qquad z = a\frac{\phi'}{\mathfrak{H}}, \tag{4.7}$$

and $\mathfrak{H} = a'/a$. The mode perturbations $\{v_k\}$ of the scalar field are the solutions of the Mukhanov-Sasaki equation [158]

$$v_k'' + \left(k^2 + \frac{z''}{z}\right) v_k = 0. \tag{4.8}$$

The conjugated momenta from (4.6) are

$$p_a = -a', \qquad p_\phi = a^2\phi', \qquad p_{v_k} = v_k^{*'}. \tag{4.9}$$

Thus, the Hamiltonian reads

$$H = \frac{1}{2}\left[-p_a^2 + 2a^4\rho(\phi) + \sum_k \left(p_{v_k} p_{v_k^*} - \omega_k^2 v_k v_k^*\right)\right], \tag{4.10}$$

where we keep the scalar field as classical whose energy density is given by Eq. (2.12). The corresponding WDW equation is found after the quantization of the operators (2.40) as

$$\left[\frac{1}{a}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) + 2a^4\rho(\phi) + \sum_k \left(-\frac{\partial^2}{\partial v_k^2} + \omega_k^2 v_k^2\right)\right] \Psi(a, \{v_k\}) = 0, \tag{4.11}$$

where the wave function of the universe is a function of the scale factor and the perturbations of the scalar field.

In order to create the third quantized scenario, I look for the action whose equation of motion for a certain field $\Psi^\dagger$ is given by Eq. (4.11), and reads

$$S_{3Q} = \int \mathrm{d}a \prod_k \mathrm{d}v_k a \left[-\frac{\partial \Psi^*}{\partial a}\frac{\partial \Psi}{\partial a} + 2a^4\rho(\phi)\Psi^*\Psi + \sum_k \left(\frac{\partial \Psi^*}{\partial v_k}\frac{\partial \Psi}{\partial v_k} + \omega_k^2 v_k^2 \Psi^*\Psi\right)\right]. \tag{4.12}$$

Hence, the associated momentum to the universe is

$$P_\Psi = -a\frac{\partial \Psi}{\partial a}, \tag{4.13}$$

with which I get the third quantized Hamiltonian of each single universe as

$$H_{3Q} = a\left[-\frac{1}{a^2}P_\Psi P_{\Psi^*} - 2a^4\rho(\phi)\Psi^*\Psi - \sum_k \left(\frac{\partial \Psi^*}{\partial v_k}\frac{\partial \Psi}{\partial v_k} + \omega_k^2 v_k^2 \Psi^*\Psi\right)\right]. \tag{4.14}$$

The total Hamiltonian is then constructed as the sum of each individual contribution and the interaction:

$$H_{\text{pair}} = H^{(1)}_{3Q} + H^{(2)}_{3Q} - a\lambda^2(a,H)(\Psi_2 - \Psi_1)^*(\Psi_2 - \Psi_1), \tag{4.15}$$

where the interaction has been taken as in Eq. (4.3), and $H^{(i)}_{3Q}$ is the Hamiltonian (4.14) for each single universe. Even this equation is familiar and quite elegant, it is convenient to play a bit with it since a brutal simplification can alleviate some future tricky calculations.

The simplification starts by taking a mode expansion of the wave function and its momentum like

$$\Psi_j = \frac{1}{\sqrt{N}}\sum_K \exp\left\{-\frac{2\pi \mathrm{i} Kj}{N}\right\}\widetilde{\Psi}_K, \qquad P_{\Psi_j} = \frac{1}{\sqrt{N}}\sum_K \exp\left\{\frac{2\pi \mathrm{i} Kj}{N}\right\}\widetilde{P}_{\widetilde{\Psi}_K}, \tag{4.16}$$

where N is the number of samples, that in our case is just two, so it simplifies to

$$\Psi_{1,2} = \frac{1}{\sqrt{2}}\left(\widetilde{\Psi}_2 \mp \widetilde{\Psi}_1\right), \qquad P_{\Psi_{1,2}} = \frac{1}{\sqrt{2}}\left(\widetilde{P}_{\widetilde{\Psi}_2} \mp \widetilde{P}_{\widetilde{\Psi}_1}\right). \tag{4.17}$$

Substitution of (4.17) into (4.15) yields

$$\widetilde{\mathcal{H}}^{\text{pair}}_{3Q} = \widetilde{\mathcal{H}}^1_{3Q} + \widetilde{\mathcal{H}}^2_{3Q} - 2a\lambda^2(a,H)\widetilde{\Psi}^*_1\widetilde{\Psi}_1, \tag{4.18}$$

which is the third quantized Hamiltonian of each mode of the total wave function, where

$$\widetilde{\mathcal{H}}^l_{3Q} = a\left[-\frac{1}{a^2}\widetilde{P}_{\widetilde{\Psi}_l}\widetilde{P}^*_{\widetilde{\Psi}_l} - 2a^4\rho(\phi)\widetilde{\Psi}^*_l\widetilde{\Psi}_l - \sum_k\left(\frac{\partial\widetilde{\Psi}^*_l}{\partial v_k}\frac{\partial\widetilde{\Psi}_l}{\partial v_k} + \omega_k^2 v_k^2\widetilde{\Psi}^*_l\widetilde{\Psi}_l\right)\right], \tag{4.19}$$

with $l = \{1,2\}$. Coming back to the parametrization $\alpha = \ln(a)$, the WDW equation for $\tilde{\Psi}_l$ is then

$$\left[\mathrm{e}^{-2\alpha}\frac{\partial^2}{\partial\alpha^2} + 2\mathrm{e}^{4\alpha}\rho(\phi) - \frac{\partial^2}{\partial v_k^2} + \omega_k^2 v_k^2 + 2\lambda^2(a,H)\delta(l-1)\right]\tilde{\Psi}_l(\alpha,\{v_k\}) = 0, \tag{4.20}$$

where $\delta(x - x_o)$ is the Dirac delta. Finally, this is the equation of motion of $\tilde{\Psi}_l(\alpha,\{v_k\})$ in which the interaction with the other universe is explicitly shown. The simplification of the interaction term as in Eq. (4.18) shows that only one of the two modes is affected by the interaction. There is no problem if the interaction appears for a single mode only. As we know, the wave function can be recovered as the inverse of the mode expansion with a certain weight for each mode. It makes completely compatible for us to be living in any of those two modes or a mixture of them. Let me then reduce the analysis to the affected one $l = 1$. I will remove the index l from now on for that reason.

Looking at Eq. (4.20), it is obvious that the perturbations $\{v_k\}$, at least, make the problem unsolvable in general. One of the solutions to this problem is to use a semiclassical ansatz like

$$\widetilde{\Psi}(\alpha, \{v_k\}) \propto \mathrm{e}^{\mathrm{i}S_0(\alpha)} \prod_k \psi_k(\alpha, \{v_k\}), \tag{4.21}$$

assuming that the $\{v_k\}$ are really small perturbations over a defined background whose contribution is here given by the function $S_0(\alpha)$. Introducing (4.21) into (4.20), we arrive to an equation where several assumptions, and then approximations, can be done.

The first is the assumption that no backreaction appears from the perturbations, it is, the background is only defined by the non-perturbative degrees of freedom. It implies that the equation of motion of the background is completely independent on $\{v_k\}$. Doing that, the background dynamics is given by the Hamilton-Jacobi equation

$$-\mathrm{e}^{-2\alpha}\left(\frac{\partial S_0}{\partial \alpha}\right)^2 = -2\mathrm{e}^{4\alpha}\rho(\phi) - 2\lambda^2(a, H), \tag{4.22}$$

from where we recognize, coming back again to the usual scale factor, that

$$\frac{\partial S_0}{\partial a} = p_a = -a\frac{\mathrm{d}a}{\mathrm{d}t}. \tag{4.23}$$

Entering this results into Eq. (4.22), one recovers the Friedmann equation

$$H^2 = 2\rho(\phi) + 2\frac{\lambda^2(a, H)}{a^4}, \tag{4.24}$$

where the contribution of the twin universe is presented as a new term.

As for the perturbations, one could assume that the phases of the modes k are summed incoherently, and that

$$\left|\frac{\partial S_0(\alpha)}{\partial \alpha}\right| \gg \left|\frac{1}{\psi_k(\alpha, \{v_k\})}\frac{\partial \psi_k(\alpha, \{v_k\})}{\partial \alpha}\right|. \tag{4.25}$$

Repeating the argument that the perturbations do not affect the background and recognizing the WKB conformal time as

$$\frac{\partial}{\partial \eta} = -\mathrm{e}^{-2\alpha}\frac{\partial S_0(\alpha)}{\partial \alpha}\frac{\partial}{\partial \alpha}, \tag{4.26}$$

one can conclude that the dynamics of the perturbations is given by

$$\mathrm{i}\frac{\partial}{\partial\eta}\Psi_k(\alpha,\{v_k\}) = \frac{1}{2}\left[-\frac{\partial^2}{\partial v_k^2} + \omega_k^2(\eta)v_k^2\right]\Psi_k(\alpha,\{v_k\}), \tag{4.27}$$

which is the Schrodinger equation (2.14).

After those considerations I am provided with enough tools to find the spectrum of the CMB. There is, however, another final consideration to be done. The interaction with the other universe is here described by the term $\lambda^2(a,H)$ dominating at the early stages since $\lambda^2(a,H)$ must be smaller than a^4 to fulfill the condition (4.5). But, as explained in Section 3.10, the entanglement and thus the interaction must be suppressed at late times. Our model up to now has never taken into account external interactions which force the system to undergo decoherence effects. That is why I consider that, after inflation, inasmuch standard cosmology is recovered, the transfer functions and the dynamics are the standard ones, abandoning the existence of our twin universe.

4.3 Observational Imprints on the CMB Spectrum

The new dynamics of the universe is at anytime ruled by Eq. (4.24), where $\lambda^2(a,H)$ is a coupling function which controls the interaction and which is inevitably small compared with the late time values of the other contributions to the Friedmann equation. The shape of $\lambda^2(a,H)$ is crucial to test the existence of the multiverse. The big problem is that we cannot know its shape in any way. I cannot do more than imposing a function which fulfills conditions (4.1) and (4.5) simultaneously.

I take under consideration a simple, but pretty general function like

$$\lambda^2(a,H) = \frac{\lambda_o}{2}\frac{a^q}{H^n} = \frac{\lambda_o}{2}\frac{a^{q+n}}{\dot{a}^n}, \tag{4.28}$$

where $n > 0$ and q are some real constants, and λ_o is a coupling constant which is small enough to see the interaction as a perturbation at late times, whose units are $[T^{-2-n}]$. The condition (4.1) is ensured since $n > 0$, and (4.5) if $q + n < 4$. Hence, the interaction dominates at early times and the perturbative method is not suitable there.

The power spectrum of the CMB is recovered with an almost de Sitter expansion during inflation. Hence it is necessary to introduce a scalar field whose potential is almost constant, or equivalently, whose barotropic parameter (3.27) is very close to $\omega_\Lambda = -1$. Thus, I define

$$\omega = -1 + \frac{2\alpha}{3}, \qquad \alpha \gtrsim 0, \tag{4.29}$$

so the density (3.18) goes like

$$\rho(a) = \frac{1}{2} H_{\rm dS}^2 \left(\frac{a_d}{a}\right)^{2\alpha}, \tag{4.30}$$

where $H_{\rm dS}$ is the Hubble parameter for a perfect de Sitter expansion (when $\alpha = 0$ and $\lambda_o = 0$), and a_d is a constant to be determined. The Friedmann equation (4.24), including the scalar field, is written as

$$H^2 = H_{\rm dS}^2 \left[\left(\frac{a_d}{a}\right)^{2\alpha} + \frac{\lambda_o}{H_{\rm dS}^2 H^n a^{4-q}} \right]. \tag{4.31}$$

The emergence of the Hubble parameter in the denominator of the term given by the interuniversal interaction makes the difference with previous studies [20].

Eq. (4.31) is a hard-to-solve differential equation. Writing it in terms of the scale factor explicitly reads

$$\dot{a}^{n+2} = H_{\rm dS}^2 \dot{a}^n a_d^{2\alpha} a^{2(1-\alpha)} + \frac{\lambda_o}{a^{2-q-n}}. \tag{4.32}$$

This way a family of functions (4.28), which satisfies that $q + n = 2$ and so the condition (4.5) for any values of n and q too, simplifies the differential equation to

$$\dot{a}^{n+2} = H_{\rm dS}^2 \dot{a}^n a_d^{2\alpha} a^{2(1-\alpha)} + \lambda_o. \tag{4.33}$$

This is an oversimplification of the system we make to get simple analytical results. Among all possible values we will consider the case $n = 1$ and $q = 1$ because it yields an extreme case of all possible dynamics. To see what I mean, one should notice that at the very early universe the scale factor scales like

$$a(t) \sim \sqrt[n+2]{\lambda_o} t, \tag{4.34}$$

where the first term of the right hand side of Eq. (4.33) is neglected. With that initial motion of the universe I find that $aH \sim \sqrt[n+2]{\lambda_o} \sim k$, which is constant. Therefore, the modes of the fluctuations do not cross the Hubble horizon. Thermal equilibrium is expected and so is a little change to the visible power spectrum. Even if I restore the neglected term in Eq. (4.33) using (4.34) as feedback, the dynamics is slightly accelerated, for which aH increases and any suppression of the spectrum is expected. The accelerated motion is increased for any other values of n and q. For that reason, considering models with coupling functions into the family for which the Friedmann equation is (4.33), the general shape of the spectrum obtained by Planck must be recovered solely by the scalar field, and the coupling constant can be taken as a perturbation at any time.

Summarizing, as a limiting example, let me consider the model where the always positive Hubble parameter contributes the most to the interaction term in the Friedmann equation (4.31), i. e., $n = 1$ and $q = 1$. For bigger n, the interaction term is smaller, so any effect obtained from this case will be suppressed. Thus, Eq. (4.31) can be expanded at first order around λ_o yielding

$$H \approx H_{\rm dS}\left(\frac{a_d}{a}\right)^{\alpha} + \frac{\lambda_o}{2H_{\rm dS}Ha^{3-\alpha}a_d^{\alpha}} \approx H_{\rm dS}\left(\frac{a_d}{a}\right)^{\alpha} + \frac{\lambda_o}{2H_{\rm dS}^2a^3}. \tag{4.35}$$

Trying an ansatz for the scale factor like

$$a(t) = a_o(t)\left[1 + \xi(t, \lambda_o)\right], \tag{4.36}$$

where a_o is the solution of Eq. (4.35) which neglects the perturbative term, and assuming that $\xi(t, \lambda_o) \ll 1$ and all second derivatives are neglected, we find the solution to (4.35) as

$$a(t) \approx a_d\left(\alpha H_{\rm dS}t\right)^{1/\alpha}\left[1 - \frac{\lambda_o}{6a_d^3H_{\rm dS}^{2+3/\alpha}(\alpha t)^{-1+3/\alpha}}\right]. \tag{4.37}$$

What is left now is the usual procedure to find the power spectrum

$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\frac{|v_k|^2}{z^2}, \tag{4.38}$$

and the angular power spectrum of the CMB. In order to do so, I find the slow-roll parameter [156]

$$\epsilon = -\frac{\dot{H}}{H^2} \approx \alpha - \lambda_o\frac{(3-\alpha)(3-2\alpha)(1-\alpha)}{6H_{\rm dS}^2a^2(\eta)}\eta, \tag{4.39}$$

and use it to write the Mukhanov-Sasaki equation (4.8), at first order in λ_o, as

$$v_k''(\eta) + \left(k^2 - \frac{2}{\eta^2} - \frac{3\alpha}{\eta^2} + \lambda_o\frac{\left[a_d^{\alpha}H_{\rm dS}(1-\alpha)\right]^{\frac{2}{1-\alpha}}}{6H_{\rm dS}^2}|\eta|^{\frac{1+\alpha}{1-\alpha}}\right)v_k(\eta) = 0, \tag{4.40}$$

where we have included

$$\eta \approx -\frac{t\alpha}{(1-\alpha)a(t)} \approx -\frac{1}{(1-\alpha)aH}, \qquad \frac{z''}{z} = (aH)^2(2-\epsilon). \tag{4.41}$$

One confronts here the problem of the boundary conditions at the beginning of inflation. Even if λ_o is taken as a perturbation, one cannot recover the Bunch-Davies vacuum at early times for this case reading

$$v_k(\eta) = \frac{\mathrm{e}^{-ik\eta}}{\sqrt{2k}}. \tag{4.42}$$

Nonetheless, I propose to consider that the classical evolution started at the very early stages, where the interaction dominates such that one can impose the Bunch-Davies vacuum at the time

$$\eta_{\text{knee}} \sim -\frac{1}{(1-\alpha)\sqrt[3]{\lambda_o}} \sim -\frac{1}{\sqrt[3]{\lambda_o}}, \tag{4.43}$$

which is the corresponding conformal time where the almost de Sitter expansion of the late times fulfills the condition that $aH = \sqrt[3]{\lambda_o}$, which is what I found as the less energetic mode inside the horizon at the early stages. Ergo, a solution can be

$$v_k(\eta) = v_k^{(0)}(\eta)\mathrm{e}^{f_k(\eta,\lambda_o)}, \tag{4.44}$$

where $v_o^{(0)}(\eta)$ is the solution to the non-perturbed Mukhanov-Sasaki equation (4.40), and $f(\eta, \lambda_o)$ is a slow varying function such that the exponential is very close to unity, and whose second order derivatives can be neglected. Thus, introducing the ansatz (4.44) into (4.40) one finds that

$$v_k(\eta) = v_k^{(0)}(\eta)\exp\left[-\lambda_o \frac{[a_d^\alpha H_{\text{dS}}(1-\alpha)]^{\frac{2}{1-\alpha}}}{12H_{\text{dS}}^2}\int^\eta |\tilde{\eta}|^{\frac{1+\alpha}{1-\alpha}}\frac{v_k^{(0)}(\tilde{\eta})}{v_k'^{(0)}(\tilde{\eta})}\mathrm{d}\tilde{\eta} + \Delta_k\right], \tag{4.45}$$

where Δ_k is a constant of integration. In consequence of the imposition of the boundary conditions at very early times, so that η_{knee} is considerably large, such constant Δ_k is almost purely imaginary, what contributes into $v_k(\eta)$ as a phase which is not going to be relevant since the power spectrum (4.38) is only dependent on its modulus.

The solution $v_k^{(0)}(\eta)$ in Eq. (4.45) is well-known to be [20, 156, 157]

$$v_k^{(0)}(\eta) = \frac{\sqrt{\pi|\eta|}}{2}H_\mu^{(1)}(k|\eta|), \qquad \mu \approx \frac{3}{2} + \frac{5}{9}\alpha, \tag{4.46}$$

where $H_n^{(1)}(z)$ is the Hankel function of the first kind.

Finally, the power spectrum (4.38) has the form:

$$P_{\mathcal{R}}(k \lesssim aH) \approx \left[(1-\alpha)\frac{\Gamma(\nu)}{\Gamma(3/2)}\frac{H}{2\pi}\right]^2 \left(\frac{k|\eta|}{2}\right)^{-2\alpha} \left|\mathrm{e}^{f_k(\eta,\lambda_o)}\right|^2, \tag{4.47}$$

which is found, in our case, at first order in $\epsilon \sim \alpha$, and at the moment when the modes crosses the horizon $k_* \approx a_* H_{\mathrm{dS}}$, to be like

$$P_{\mathcal{R}}(k \lesssim aH) \approx \frac{1}{2^{-2\alpha}}\left\{\left[1-\alpha+\Psi\left(\frac{3}{2}\right)\alpha\right]\frac{H_{\mathrm{dS}}}{2\pi}\right\}^2 \left(\frac{k}{a_* H_{\mathrm{dS}}}\right)^{-2\alpha} \left|\mathrm{e}^{f_k(\eta,\lambda_o)}\right|^2, \tag{4.48}$$

where $\Psi(z) := \Gamma'(z)/\Gamma(z)$ is the digamma function.

The power spectrum (4.38) is expected to fit the power law

$$P_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1}, \tag{4.49}$$

when $aH \sim k$, or equivalently, $k|\eta| \sim 1$, where A_s, and n_s depend on the given value k_* set by the experiment. From the Planck analysis [12], where they considered $k_* = 0.05$ Mpc^{-1}, it was obtained $A_s = (2.105 \pm 0.030) \times 10^{-9}$, and $n_s = (0.9665 \pm 0.0038)$. The absolute value of the exponential can be considered very close to unity such that we can derive, comparing with the fit (4.49), the values

$$\alpha = 0.0168 \pm 0.0019, \qquad a_d = a_* = (9.052 \pm 0.067) \cdot 10^{-56}, \tag{4.50}$$

$$H_{\mathrm{dS}} = (2.896 \pm 0.021) \cdot 10^{-4} = (5.372 \pm 0.039) \cdot 10^{39}\,\mathrm{s}^{-1}. \tag{4.51}$$

I would like to point out that the actual values, in dimensionless units, are

$$a_0 = 1, \qquad H_0 \approx 1.2 \cdot 10^{-61}, \tag{4.52}$$

for comparison.

Lastly, the power spectrum (4.38) and the solution (4.45) are used to find the angular power spectrum, given in terms of the coefficients

$$C_l = 2T_0^2 l(l+1)\int_0^\infty \frac{\mathrm{d}k}{k} P_{\mathcal{R}}(k)\Delta_l^2(k), \tag{4.53}$$

where $T_0 = (2.72548 \pm 0.00057)$ K is the temperature of the CMB [159], and CAMB has been used to obtain the transfer functions $\Delta_l(k)$. The power spectrum and the angular power

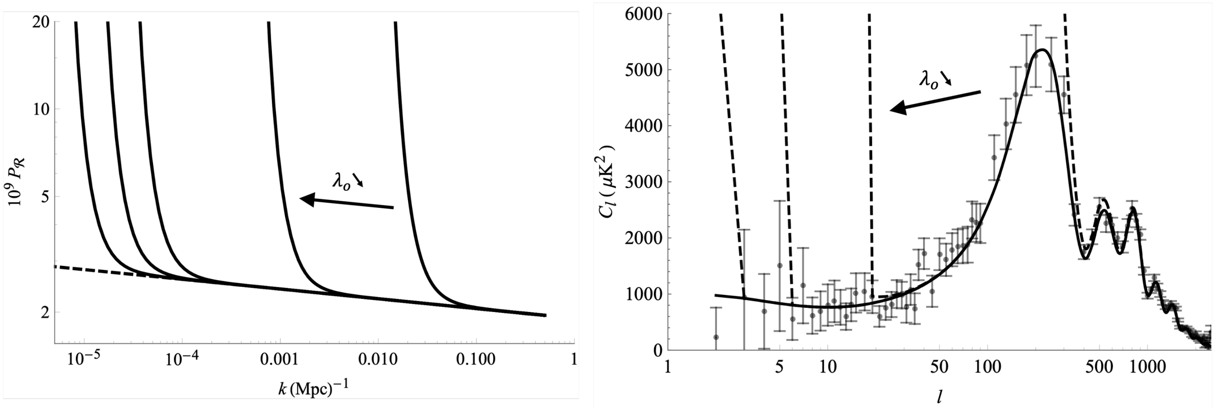

Figure 4.1: For both figures, the considered values for λ_o are: 10^{-46}, 10^{-50}, 10^{-54}, 10^{-55} and 10^{-56} s^{-3}. Left panel: The power spectrum for different values of λ_o is shown. The dashed line represents the known spectrum (4.49) with the values given by Planck. We find that the smaller the value of λ_o, the more similar the spectrum to the standard, as expected. The effect of the interaction affects only to the smaller modes k. Right panel: The normalized angular power spectrum is shown for different values of λ_o. The solid line represents the standard spectrum. The points and the error bars come from the data measured by Planck. The dashed lines are the spectrum found for different λ_o. For $\lambda_o = 10^{-56}\,\mathrm{s}^{-3}$, the dashed line is overlapped by the solid line. Here, the smaller λ_o, the smaller are the affected multipoles.

spectrum of the CMB are shown in Figure 4.1. Both spectra show the same conclusions. The interaction affects them as much as the coupling constant λ_o grows. It is about

$$\lambda_o \sim \mathcal{O}(10^{-56})\,\mathrm{s}^{-3}, \tag{4.54}$$

when the whole spectrum is fully recovered. The spectrum is enlarged very rapidly for those k or multipoles which are affected by the interaction. Thus, it is very unlikely that the value of λ_o is bigger than 10^{-56}, so it would be seen at the low multipoles of the angular power spectrum very easily. Unluckily, that is also the reason why the CMB angular spectrum does not look like a good observation to check the existence of a twin universe, but to find an upper bound for the coupling constant of the interaction.

This was the most extreme case in which the interaction is the largest. If n into Eq. (4.35) would have been taken larger, the constant λ_o would have been much smaller.

However, it is just a matter of fixing constants and the coupling function $\lambda^2(a,H)$ into the Friedmann equation (4.24). The procedure would be the same, and the results on the spectra identical to the ones obtained here.

Another coupling functions $\lambda^2(a,H)$ could be considered in order to take into account exotic singularities. Those require a more advanced ansatz which could contain higher derivatives of the scale factor as jerk, snap and pop [160, 161]. Let us define a generalization of my coupling function (4.28) like

$$\lambda^2(a,H,\mathcal{X}) = \frac{\lambda_o}{2}\frac{a^q}{H^n\mathcal{X}^m} = (-1)^m\frac{\lambda_o}{2}\frac{a^{q+n-m}}{\dot{a}^{n-2m}\ddot{a}^m}, \tag{4.55}$$

where $\mathcal{X} = -(a\ddot{a})/\dot{a}^2$, and $m > 0$ is a constant. In order to fulfill the conditions (4.5) and to reduce the Friedmann equation as I did to arrive to Eq. (4.33), the constraints on the constants are $q + n < 4 + m$ and $q + n = 2 + m$. The analysis is analogous to the one I did before, and no relevant new feature appears.

For instance, let us consider $m = 2$, $n = 4$ and $q = 0$, so the Friedmann equation (4.24) is written like

$$H^2 = H_{\rm dS}^2\left[\left(\frac{a_d}{a}\right)^{2\alpha} + \frac{\lambda_o}{H_{\rm dS}^2 a^2\ddot{a}^2}\right]. \tag{4.56}$$

At the initial singularity, the scale factor goes like $t^{3/2}$, which corresponds to the evolution of a perfect fluid whose barotropic parameter is $\omega = -5/9$, and at late times it goes like $t^{1/\alpha}$, where $\alpha << 1$. From the very beginning until the end of inflation, the content of the universe could be parametrized by a scalar field whose barotropic parameter changes as the universe evolves from $-5/9$ to -1. It is an always expanding universe for which all conclusions given for the previous model are admissible. Therefore, the spectra would be slightly changed in every case. As it was an example, the analysis of the imprints on the CMB has gone too far already in this specific case.

4.4 Comment about an Interuniversal Contribution to the Dark Energy

To finish, let me show the case which looks like an academic curiosity more than a realistic scenario. It is the one where the coupling function $\lambda^2(a, H)$ in Eq. (4.24) to be of the form

$$\lambda^2(a, H) = \frac{\lambda_o}{2} a^4 f(H) H^2, \tag{4.57}$$

where $f(H)$ is an explicit smooth function of H and λ_o is a positive constant. In order to satisfy the condition (4.1), it is enough that $f(H)H^2$ diverges when H vanishes. The Friedmann equation (4.24), without any scalar field, i. e. with $\rho(\phi) = 0$, yields the trivial result

$$H = f^{-1}(\lambda_o^{-1}) = \text{constant}, \tag{4.58}$$

so the dynamics is the one of a de Sitter universe, but the one which has not got the cosmological constant.

I say that this is a purely academic example for some reasons. For example, first, the probability of the vacuum to decay (4.4) goes like

$$\Gamma \propto \exp\left\{ - \left[\lambda_o f(H) H^2 \right]^{-3} \right\}, \tag{4.59}$$

which is constant during the whole evolution of the universe. That is not safe for the stability of the false vacuum regions. Furthermore, there is no way to reproduce the spectrum of the CMB without a field which is already there before inflation, so it contributes to the matter content of the universe, enters the Friedmann equation (4.24), and modifies the constancy of (4.58). Hence, in principle, unless there is a different origin for the primordial fluctuations seen at the CMB spectrum, this will remain as an academic curiosity.

5

New Interpretation of the Third Quantization Scheme

The question to solve in this short chapter is simple and very important: does it make any sense what we have got so far? And with this question I do not refer to the research presented in this text assuming that the theoretical background is well-substantiated. Whether the results I got are correct or not is a matter of cut and thrust between yet different outstanding points of view of the multiverse. Until we have got plain observational verification of any theory of the multiverse, all research on the field have been somehow reasonable. However, the question is raised regarding the most fundamental concepts of the 3rdQ theory. Also, whether CQG is correct is another question which has lasted for several decades already. All problems of CQG partially listed in Section 2.2 are well-known, but with no solutions. For now, let just assume that CQG is solid enough.

The field theory of universes intended by 3rdQ of CQG was the conducting theory used to find the EE of a pair of universes, in analogy with QED, for example. In principle, the action (2.65) describes the interactions in the multiverse, where the field Ψ is, in principle, the only particle in the multiverse. Two particles travelling through a spacetime in which no other particles exist, cannot be modified in any sense. Indeed, the only interaction allowed is described by the potential term, given by a certain external field, where $m^2(\alpha, \phi)$ enters, and it cannot change the nature of the field Ψ unless it feels another particle $\Psi^\dagger$ and releases... What particle? What is the vertex from the action (2.65)? Is it well defined? The obvious answer is no.

The analogy with KG equation (2.18) was done very naively [162]. The Lagrangian of a field theory does not contain a function $m^2(\alpha, \phi)$ which is not described by a field, or the vacuum expectation of a field which indeed would be constant, as it happens with the Higgs mechanism [1, 2]. There is not real relation between the KG equation (2.18) and the WDW equation (2.53) since the masses of both of them are essentially different. There is a constant, which makes the theory treatable, and the other is a function of the *internal* degrees of freedom of the universes, that in the minisuperspace are the scale factor and the scalar field. Therefore, considering the scale factor as a time variable, the mass term changes in time. Any dynamics in that direction of the arrow of time must be taken into account carefully. Regarding the pair of particles, what is shocking, is the dependence of the EE on time, that should not be there since there is not any kind of environment assumed. The nature of such a dependence must be understood as a fundamental incompleteness of the description of the theory and its treatment.

Besides, it is interesting to remark that the action (2.65) is not bounded from below if $m^2(\alpha, \phi)$ in Eq. (2.54) is negative. Even though, this argument is quite weak since $\Psi^\dagger\Psi$ is not related to any probabilistic interpretation of QM.

The way a field theory should modify 3rdQ, such that the EE of the pair moving through a supposedly empty space is understood, as I found it, by introducing a new field Ω accounting for the variability of $m^2(\alpha, \phi)$ in the WDW equation (2.53), which acts as a source term. Taking a step back by regarding the most general superspace, this new field would be the cause of the dynamics of the potential term of Eq. (2.41) proportional to

$$\sqrt{h}\left[-{}^{(3)}R + 2\Lambda + 16\pi(K_s + V)_{\text{matter}}\right] \supset \mathcal{H}, \tag{5.1}$$

where K_s and V are the kinetic part of the spatial coordinates of the matter content and V its potential energy.

There are many actions for which the WDW equation (2.53) is an equation of motion for $\Psi^\dagger$ through the Euler-Lagrange equations

$$\frac{\partial}{\partial\alpha}\left(\frac{\delta\mathcal{L}}{\delta(\partial_\alpha\Psi^\dagger)}\right)+\frac{\partial}{\partial\phi}\left(\frac{\delta\mathcal{L}}{\delta(\partial_\phi\Psi^\dagger)}\right)-\frac{\delta\mathcal{L}}{\delta\Psi^\dagger}=0. \tag{5.2}$$

For example, let us consider a real scalar field Ω. The action is then

$$S_{3Q}=\int \mathrm{d}\alpha\mathrm{d}\phi\left[-\Psi^\dagger\Box\Psi-\frac{1}{2}\Omega\Box\Omega+g_3\Omega\Psi^\dagger\Psi+V(\Omega^2)\right], \tag{5.3}$$

where $V(\Omega^2)$ is the particular potential of the new field, g_3 is a coupling constant and we recognize $g_3\Omega = m^2$. The field Ω is expected to be real since it is proportional to $m^2(\alpha,\phi)$, which can be any real, possibly negative, number. Thus, the new field has its own dynamics ruled by

$$-\frac{1}{g_3}\left\{\left[\frac{\partial^2}{\partial\alpha^2}-\frac{\partial^2}{\partial\phi^2}\right]\Omega+\frac{\partial V(\Omega^2)}{\partial\Omega}\right\}=\Psi^\dagger\Psi. \tag{5.4}$$

From this equation of motion it seems that there is a way to control the unitarity of the wave function, since there is an explicit expression for $|\Psi|^2$ given in terms of Ω, but such control is not perfect. For instance, let us consider that the wave function of the universe fulfills $|\Psi|^2 = C$ at all points of the phase space, where C is a positive constant, and a potential $V(\Omega^2) = M_\Omega\Omega^2/2$. Eq. (5.4) is then

$$\left\{\left[\frac{\partial^2}{\partial\alpha^2}-\frac{\partial^2}{\partial\phi^2}\right]\Omega+M_\Omega\Omega\right\}=-g_3. \tag{5.5}$$

Taking a look at Eq. (2.54), the limit at early times shows a vanishing function for $m^2(\alpha,\phi)$ and its derivatives, hence Ω is also vanishing. It is not in agreement with what we have obtained from Eq. (5.5) since Ω cannot be vanishing. Therefore, there is no way to keep a constant $|\Psi|^2$ at least with the potential we used.

Another natural example is to consider a complex scalar field Ω. The action then reads

$$S_{3Q}=\int \mathrm{d}\alpha\mathrm{d}\phi\Big[-\Psi^\dagger\Box\Psi-\Omega^\dagger\Box\Omega+g_4\Omega^\dagger\Omega\Psi^\dagger\Psi+V(|\Omega|^2)\Big], \tag{5.6}$$

from where we recognize that $m^2 = g_4\Omega^\dagger\Omega > 0$. It is important to notice that here the action is bounded from below, since $m^2(\alpha,\phi) > 0$, which was our weak condition to get a bounded action (2.65). It is not a problem to describe the actual universe as the mass term $m^2(\alpha, \phi)$ has always been positive during standard ΛCDM evolution. The dynamics of the new field $\Omega^\dagger$ is now given by

$$\left[\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} + g_4\Psi^\dagger\Psi\right]\Omega = -\frac{\partial V(|\Omega|^2)}{\partial\Omega^\dagger}, \tag{5.7}$$

while Ω obeys the complex conjugated equation. Both equations for the new particle are similar to a WDW equation, and coincide when $V(|\Omega|^2)$ vanishes. Whether self-interactions of the Ω field exist or not is impossible to say, and that is why we cannot conclude that it is a universe-like field as Ψ. Even so, it keeps intentionally the symmetry $U(1)$ found in the action (2.65) in order to get the total electric charge still conserved, and besides, the number of particles. This example is clearly more natural than the first one. However, there is not a three-leg vertex predicted by the latter model as in the former one, which makes the extensively-studied picture of parent universes giving birth to baby universes [87, 88] impossible, unless exotic ingredients like axion-like particles are included [74].

Nonetheless, most of the work done on 3rdQ is not spoiled, since the WDW equation (2.53) may be understood as an effective description of the underlying field theory we propose. Note that due to the "coordinate" dependence of the mass in the third quantized action (2.65), a nonperturbative solution of the path integral was not achievable in the first place, seemingly resulting in a hidden Fermi-like interaction provided on the left of Figures 5.1 and 5.2. These diagrams are completed by the introduction of the real scalar and the complex scalar mediators to the right of Figures 5.1 and 5.2, respectively.

For instance, the entanglement entropy of the pair of universes calculated in Refs. [111, 121, 137] remains unchanged for all models. However, the spectra of the infinite universes created by dynamical variables in minisuperspace [83–85] square well with this new interpretation because the background is flat, while an external field is held accountable for this variation. The resulting field theory on flat spacetime, including a source term, is thus equivalent to the canonical 3rdQ formalism on a dynamical, i. e. expanding, minisuperspace. The only difference lies in the interactions between universes and antiuniverses by virtue of a new field. Hence, the concept of 3rdQ is carried through all the way inasmuch as every field is quantized instead of treating some of them as classical background.

I leave this new direction of research open for the future, but it is just a reinterpretation of what we already knew about 3rdQ picture; just two faces of the same coin. Most of the observational studies has been done based on the WDW equation without using 3rdQ or using the WDW equation as an effective equation of motion of the universes. It yields the same dynamics on our universe and therefore the results are the same, as it happens with all the results obtained in the previous chapters.

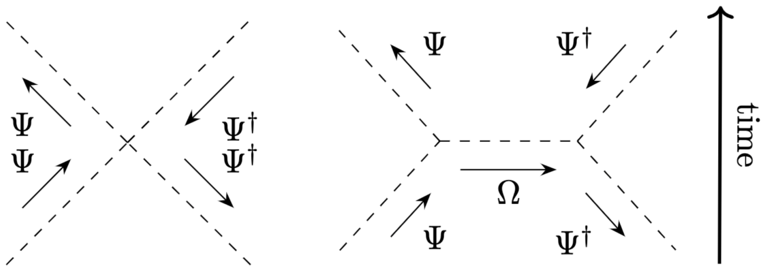

Figure 5.1: Interaction between universes without the mediation by the Ω-bosons (left) and with them (right) as derived from the action (5.3), the second example. Time's flow implies a change in the variable α, the scale factor.

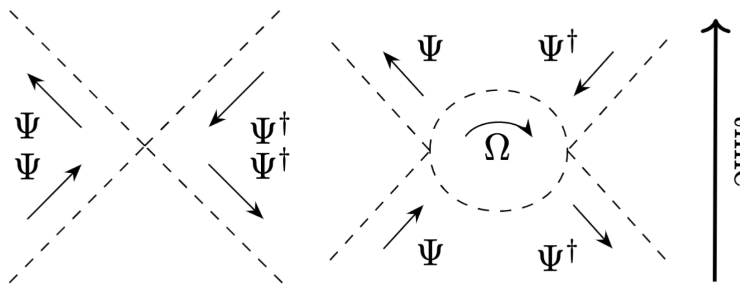

Figure 5.2: Interaction between universes without the mediation by the Ω-bosons (left) and with them (right) as derived from the action (5.6), the second example. Time's flow implies a change in the variable α, the scale factor.

6

Conclusions and Perspectives

An interacting level III multiverse in Tegmark's classification has been built on the 3rdQ formalism of CQG. In that multiverse the universes behave like particles in QFT. Due to the Schwinger effect, a pair of universes can be created at the same point of the spacetime. It is entangled and this entanglement is measured by the entanglement entropy. In order to properly calculate the entanglement, we have used the invariant representation in which the number of universes is constant and the entanglement appears well-defined.

We have considered the von Neumann entropy (2.74) as the measure of the entanglement as well as its extensions like Tsallis and Rènyi entropies (2.76). Other parameters such as the temperature of entanglement (2.79) and Q (2.80), which have been suggested before as good indicators of the entanglement, have also been studied in some cases.

Analysing many models of FLRW geometries we have obtained the following conclusions about the EE of the pair of universes:

- First and quite important, there is always a pair of solutions to describe each of the universes of the pair such that it is compatible with the pair creation process given by 3rdQ formalism. Otherwise, there could have been models for which it was impossible to express them properly.

- Any distribution function $A(k)$ like in Eq. (2.60) must be symmetric around $k = 0$ in order to easily explain the matter-antimatter asymmetry of the universe. Using this property, we consider solutions (3.7) which simplify further calculations.

- The classical phase space of each universe is recovered by finding the imaginary part of the EE of the pair. Out of the allowed region of the phase space, the entanglement entropy is not real. It happens because it has a logarithmic shape which diverges at the critical points of the classical evolution, i. e., when the Hubble parameter H vanishes.

- The EE of every model has been found divergent at any critical point. It is in perfect harmony with Kiefer's conclusions in Ref. [65], where he states that the maximum of the expansion of the universe must be a very quantum point since there, expanding and contracting phases of the universe are overlapped.

- The treatment of the scalar field has an impact on the EE at the initial singularity. A scalar field which is treated classically makes the EE divergent while treated quantum mechanically makes it finite. The reason of this regularization is not yet clear. Besides, there are only two models for which the EE is the same, being those pairs de Sitter universes or stiff matter dominated universes.

- The divergent behaviour of the EE has been also found for some exotic singularities like the Little Rip singularity, but not for other like Big Brake, Big Separation or Big Freeze singularities. It implies that it is not a particular behaviour of the EE at the critical points, but for a more general set of points of the phase space.

- The obvious dependence of the von Neumann entropy on the Hubble parameter cannot be obtained in general, but only around those critical points where it diverges. Besides, the EE was found to have a logarithmic behaviour like the Shannon information content (3.53) of an event with probability H.

- The generalized entropies like Tsallis and Rènyi entropies yield similar results to the von Neumann entropy in a general case. Besides, Rènyi entropy follows an asymptotic logarithmic shape in terms of the Hubble parameter around the critical points, but the Tsallis entropy does not.

These have been a pretty large amount of properties of the EE which we have found for a single pair of universes so far. The main conclusions of all those properties, however, are expected to hold for a more general scenario where additional interactions are imposed. The way those interactions are working in the theory is unknown and would affect the WDW equation and the dynamics of the universes. The explanation for the new interaction terms may be understood as coming from a more general theory of gravity [163, 164], including corrections to the curvature term of the Einstein-Hilbert action (2.6) or nonlocal terms.

All those considerations have been taken into account to impose a heuristic interaction, like (4.28) or (4.55), between the pair of universes and to see if we could find some hints on the CMB of our twin antiuniverse in case it exists. With some very special care, it has been found that the interaction must be very small, being the coupling constant $\lambda_o \lesssim \mathcal{O}(10^{-56})\,\mathrm{s}^{-3}$, in order to recover the spectrum we observe, and that it is affecting the spectra of the CMB (see Figure 4.1) exaggeratedly for the smaller values of k or the multipoles. In fact, the spectrum of the CMB is, at the end of the day, useless to distinguish the kind of interaction described by the coupling function $\lambda^2(a, H)$, but it is useful to constraint its strength, which is measured by the coupling constant λ_o. Hence, we need to wait for future observations as the cosmic neutrino background or even the spectrum of primordial gravitational waves to really test the model. They will provide us with relevant information about an earlier stage of the evolution.

Finally, we have offered a new perspective of the 3rdQ scenario in which it becomes a true field theory of universes, where, at least, a new particle must be included into a multiverse of this kind to be the reason of the source term $m^2(\alpha, \phi)$ which appears in the action (2.65). This way, the theory is completed as a full quantum field theory even though it is impossible to know the real Lagrangian of the fundamental theory yet.

7 References

[1] M. D. Schwartz, *Quantum Field Theory and the Standard Model* (Cambridge University Press, Mar. 2014).

[2] S. Weinberg, *The quantum theory of fields*, Vol. 1 (Cambridge University Press, 1995).

[3] B. S. DeWitt, "Quantum theory of gravity. I. The canonical theory", Phys. Rev. **160**, 1113–1148 (1967).

[4] B. S. DeWitt, "Quantum theory of gravity. II. The manifestly covariant theory", Phys. Rev. **162**, 1195–1239 (1967).

[5] B. S. DeWitt, "Quantum theory of gravity. III. Applications of the covariant theory", Phys. Rev. **162**, 1239–1256 (1967).

[6] P. A. M. Dirac, "Fixation of coordinates in the Hamiltonian theory of gravitation", Phys. Rev. **114**, 924–30 (1959).

[7] P. G. Bergmann, "Hamilton–Jacobi and Schrödinger theory in theories with first-class Hamiltonian constraints.", Phys. Rev. **144**, 1078–80 (1966).

[8] R. Arnowitt, S. Deser, and C. W. Misner, "The dynamics of general relativity", in *Gravitation: an introduction to current research*, edited by L. Witten (Wiley, 1962), pp. 227–65.

[9] J. A. Wheeler, "On the Nature of quantum geometrodynamics", Annals Phys. **2**, 604–614 (1957).

[10] M. Tegmark, "Parallel universes", in *Science and ultimate reality: quantum theory, cosmology, and complexity*, edited by J. D. Barrow, P. C. W. Davies, and C. L. Harper (Cambridge University Press, 2004), pp. 459–491.

[11] N. Caderni and M. Martellini, "Third quantization formalism for hamiltonian cosmologies", Int. J. Theor. Phys. **23**, 233–249 (1984).

[12] Planck Collaboration (Planck), "Planck 2018 results. VI. Cosmological parameters", Astron. Astrophys. **641**, [Erratum: Astron.Astrophys. 652, C4 (2021)], A6 (2020).

[13] R. Holman, L. Mersini-Houghton, and T. Takahashi, "Cosmological avatars of the landscape. I. Bracketing the supersymmetry breaking scale", Phys. Rev. D **77**, 063510 (2008).

[14] R. Holman, L. Mersini-Houghton, and T. Takahashi, "Cosmological avatars of the landscape. II. CMB and LSS signatures", Phys. Rev. D **77**, 063511 (2008).

[15] S. Kanno, "Cosmological implications of quantum entanglement in the multiverse", Physics Letters B **751**, 316–320 (2015).

[16] W. H. Kinney, "Limits on entanglement effects in the string landscape from Planck and BICEP/Keck data", Journal of Cosmology and Astroparticle Physics **2016**, 013 (2016).

[17] E. D. Valentino and L. Mersini-Houghton, "Testing predictions of the quantum landscape multiverse 1: the Starobinsky inflationary potential", Journal of Cosmology and Astroparticle Physics **2017**, 002 (2017).

[18] E. D. Valentino and L. Mersini-Houghton, "Testing predictions of the quantum landscape multiverse 2: the exponential inflationary potential", Journal of Cosmology and Astroparticle Physics **2017**, 020 (2017).

[19] E. Di Valentino and L. Mersini-Houghton, "Testing predictions of the quantum landscape multiverse 3: the hilltop inflationary potential", Symmetry **11** (2019).

[20] M. B. López, M. Krämer, J. Morais, and S. R. Pérez, "The interacting multiverse and its effect on the cosmic microwave background", Journal of Cosmology and Astroparticle Physics **2019**, 057–057 (2019).

[21] A. Einstein, "Die Feldgleichungen der Gravitation", Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften, 844–847 (1915).

[22] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W.H. Freeman and Company, 1973).

[23] B. Janssen, *Teoría de la Relatividad General* (Universidad de Granada, 2022).

[24] M. Visser and C. Barceló, "Energy conditions and their cosmological implications", in Cosmo-99, edited by U. Cotti, R. Jeannerot, G. Senjanovic, and A. Smirnov (Sept. 2000).

[25] S. Weinberg, *Cosmology* (Oxford University Press, 2008).

[26] S. W. Hawking and R. Penrose, "The singularities of gravitational collapse and cosmology", Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences **314**, 529–548 (1970).

[27] M. P. Dąbrowski, K. Marosek, and A. Balcerzak, "Standard and exotic singularities regularized by varying constants", Mem. Soc. Ast. It. **85**, edited by C. Martins and P. Molaro, 44–49 (2014).

[28] W. Heisenberg, "A quantum-theoretical reinterpretation of kinematic and mechanical relations", Z. Phys. **33**, 879–893 (1925).

[29] E. Schrödinger, "Quantisierung als Eigenwertproblem", Annalen Phys. **386**, 109–139 (1926).

[30] G. Galilei, *Two New Science*, (Originally published by Elzevir, 1638). (Dover, New York, 1954).

[31] H. Goldstein, *Classical mechanics*, 3rd ed., (Updated by Charles Poole and John Safko) (Addison-Wesley, 2001).

[32] P. A. M. Dirac, *The principles of quantum mechanics* (Clarendon Press, 1930).

[33] O. Klein, "Quantentheorie und fünfdimensionale Relativitätstheorie", Zeitschrift fur Physik **37**, 895–906 (1926).

[34] W. Gordon, "Der Comptoneffekt nach der Schrödingerschen Theorie", Zeitschrift fur Physik **40**, 117–133 (1926).

[35] N. D. Birrell and P. C. W. Davies, *Quantum fields in curved space*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, 1982).

[36] N. N. Bogoljubov, V. V. Tolmachov, and D. V. Širkov, "A new method in the theory of superconductivity", Fortschritte der Physik **6**, 605–682 (1958).

[37] A. Vilenkin, "Creation of universes from nothing", Physics Letters B **117**, 25–28 (1982).

[38] J. B. Hartle and S. W. Hawking, "Wave function of the universe", Phys. Rev. D **28**, 2960–2975 (1983).

[39] H. Everett, ""Relative state" formulation of quantum mechanics", Rev. Mod. Phys. **29**, 454–462 (1957).

[40] B. S. DeWitt, "Quantum mechanics and reality", Physics Today **23**, 30–35 (1970).

[41] J. B. Barbour, "The timelessness of quantum gravity: II. The appearance of dynamics in static configurations", **11**, 2875–2897 (1994).

[42] C. Kiefer, *Quantum gravity; 3rd ed.* International series of monographs on physics (Oxford University Press, Oxford, 2012).

[43] T. Thiemann, *Modern canonical quantum general relativity*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, 2007).

[44] T. Christodoulakis and J. Zanelli, "Operator ordering in quantum mechanics and quantum gravity", Il Nuovo Cimento B **93**, 1–21 (1986).

[45] N. Kontoleon and D. L. Wiltshire, "Operator ordering and consistency of the wave function of the universe", Physical Review D **59** (1999).

[46] J. Haga and R. L. Maitra, “Factor ordering and path integral measure for quantum gravity in (1+1) dimensions”, Symmetry, Integrability and Geometry: Methods and Applications (2017).

[47] N. C. Tsamis and R. P. Woodard, “The factor-ordering problem must be regulated”, Phys. Rev. D **36**, 3641–3650 (1987).

[48] C. J. Isham, “Conceptual and geometrical problems in quantum gravity”, in Recent aspects of quantum fields, edited by H. Mitter and H. Gausterer (1991), pp. 123–229.

[49] M. Pavsic, “How the geometric calculus resolves the ordering ambiguity of quantum theory in curved space”, Classical and Quantum Gravity **20**, 2697–2714 (2003).

[50] Y. Ohkuwa, Y. Ezawa, and M. Faizal, “Operator ordering ambiguity and third quantization”, Annals of Physics **414**, 168072 (2020).

[51] F. J. Tipler, “Interpreting the wave function of the universe”, Physics Reports **137**, 231–275 (1986).

[52] W. B. Drees, “Interpretation of the wave function of the universe”, International Journal of Theoretical Physics **26**, 939–942 (1987).

[53] A. Vilenkin, “Interpretation of the wave function of the universe”, Phys. Rev. D **39**, 1116–1122 (1989).

[54] J. B. Hartle, “Prediction in quantum cosmology”, in *Gravitation in astrophysics: cargèse 1986*, edited by B. Carter and J. B. Hartle (Springer US, Boston, MA, 1987), pp. 329–360.

[55] S. Wada, “Interpretation and predictability of quantum mechanics and quantum cosmology”, Modern Physics Letters A **03**, 645–651 (1988).

[56] C. Kiefer, “The semiclassical approximation to quantum gravity”, in *Canonical Gravity: From Classical to Quantum* (Springer Berlin Heidelberg, 1993), pp. 170–212.

[57] A. Vilenkin, “The quantum cosmology debate”, AIP Conference Proceedings **478**, 23–29 (1999).

[58] A. Vilenkin, “Boundary conditions in quantum cosmology”, Phys. Rev. D **33**, 3560–3569 (1986).

[59] C. Kiefer and P. Peter, “Time in quantum cosmology”, Universe **8** (2022).

[60] C. Kiefer, “Conceptual problems in quantum gravity and quantum cosmology”, ISRN Mathematical Physics **2013**, 1–17 (2013).

[61] K. V. Kuchař, “Time and interpretations of quantum gravity”, International Journal of Modern Physics D **20**, 3–86 (2011).

[62] T. Singh and T. Padmanabhan, “Notes on semiclassical gravity”, Annals of Physics **196**, 296–344 (1989).

[63] J. J. Halliwell, “The interpretation of quantum cosmological models”, in 13th Conference on General Relativity and Gravitation (GR-13) (1993), pp. 63–80.

[64] H. Zeh, “Time in quantum gravity”, Physics Letters A **126**, 311–317 (1988).

[65] C. Kiefer, “Wave packets in minisuperspace”, Phys. Rev. D **38**, 1761–1772 (1988).

[66] F. W. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark, *NIST handbook of mathematical functions*, 1st (Cambridge University Press, USA, 2010).

[67] M. Abramowitz and I. Stegun, *Handbook of mathematical functions: with formulas, graphs, and mathematical tables*, Applied mathematics series (Dover Publications, 1965).

[68] A. R. Liddle and D. H. Lyth, *Cosmological inflation and large-scale structure* (Cambridge University Press, 2000).

[69] V. Mukhanov and S. Winitzki, *Introduction to quantum effects in gravity* (Cambridge University Press, 2007).

[70] P. D. D’Eath, S. W. Hawking, and O. Obregon, “Supersymmetric Bianchi models and the square root of the Wheeler-DeWitt equation”, Phys. Lett. B **300**, 44–48 (1993).

[71] A. Mostafazadeh, “Two-component formulation of the wheeler–dewitt equation”, Journal of Mathematical Physics **39**, 4499–4512 (1998).

[72] N. Kan, T. Aoyama, T. Hasegawa, and K. Shiraishi, “Third quantization for scalar and spinor wave functions of the universe in an extended minisuperspace”, **(arXiv 2110.06469)** (2021).

[73] M. McGuigan, “Third quantization and the Wheeler-DeWitt equation”, Phys. Rev. D **38**, 3031–3051 (1988).

[74] S. B. Giddings and A. Strominger, “Baby universe, third quantization and the cosmological constant”, Nuclear Physics B **321**, 481–508 (1989).

[75] A. Hosoya and M. Morikawa, “Quantum field theory of the universe”, Phys. Rev. D **39**, 1123–1129 (1989).

[76] L. O. Pimentel and C. Mora, “Third quantization of Brans–Dicke cosmology”, Physics Letters A **280**, 191–196 (2001).

[77] A. Balcerzak and K. Marosek, “Emergence of multiverse in third quantized varying constants cosmologies”, The European Physical Journal C **79** (2019).

[78] L. Campanelli, “Creation of universes from the third-quantized vacuum”, Physical Review D **102** (2020).

[79] A. Balcerzak and M. Lisaj, “Decaying universes and the emergence of Bell-type interuniversal entanglement in varying fundamental constants cosmological model”, **(arXiv 2112.14166)** (2021).

[80] S. J. Robles Pérez, “Quantum cosmology with third quantisation”, Universe **7** (2021).

[81] M. McGuigan, “Universe creation from the third quantized vacuum”, Phys. Rev. D **39**, 2229 (1989).

[82] X. Yingming and L. Liao, “Third quantization of a solvable model in quantum cosmology in Brans-Dicke theory”, Chinese Physics Letters **8**, 52–55 (1991).

[83] S. A. Fulling, “Nonuniqueness of canonical field quantization in riemannian space-time”, Phys. Rev. D **7**, 2850–2862 (1973).

[84] P. C. W. Davies, “Scalar production in Schwarzschild and Rindler metrics”, Journal of Physics A: Mathematical and General **8**, 609–616 (1975).

[85] W. G. Unruh, “Notes on black-hole evaporation”, Phys. Rev. D **14**, 870–892 (1976).

[86] M. McGuigan, “Universe decay and changing the cosmological constant”, Phys. Rev. D **41**, 418–430 (1990).

[87] S. Robles Pérez and P. F. González-Díaz, “Quantum state of the multiverse”, Physical Review D **81** (2010).

[88] D. Marolf and H. Maxfield, “Transcending the ensemble: baby universes, spacetime wormholes, and the order and disorder of black hole information”, Journal of High Energy Physics **2020** (2020).

[89] J. Schwinger, “On gauge invariance and vacuum polarization”, Phys. Rev. **82**, 664–679 (1951).

[90] K. A. et al., “Inflation physics from the cosmic microwave background and large scale structure”, Astroparticle Physics **63**, Dark Energy and CMB, 55–65 (2015).

[91] H. R. Lewis and W. B. Riesenfeld, “An exact quantum theory of the time-dependent harmonic oscillator and of a charged particle in a time-dependent electromagnetic field”, Journal of Mathematical Physics **10**, 1458–1473 (1969).

[92] I. A. Pedrosa, “Comment on "Coherent states for the time-dependent harmonic oscillator"”, Phys. Rev. D **36**, 1279–1280 (1987).

[93] J.-Y. Ji, J. K. Kim, and S. P. Kim, “Heisenberg-picture approach to the exact quantum motion of a time-dependent harmonic oscillator”, Phys. Rev. A **51**, 4268–4271 (1995).

[94] H.-C. Kim, M.-H. Lee, J.-Y. Ji, and J. K. Kim, “Heisenberg-picture approach to the exact quantum motion of a time-dependent forced harmonic oscillator”, Physical Review A **53**, 3767–3772 (1996).

[95] J.-Y. Ji, J. K. Kim, S. P. Kim, and K.-S. Soh, “Exact wave functions and nonadiabatic berry phases of a time-dependent harmonic oscillator”, Phys. Rev. A **52**, 3352–3355 (1995).

[96] H. Kanasugi and H. Okada, “Systematic treatment of general time-dependent harmonic oscillator in classical and quantum mechanics”, Progress of Theoretical Physics **93**, 949–960 (1995).

[97] S. R. Pérez, “Invariant vacuum”, Physics Letters B **774**, 608–615 (2017).

[98] S. Abe, “Fluctuations around the Wheeler-DeWitt trajectories in third-quantized cosmology”, Phys. Rev. D **47**, 718–721 (1993).

[99] M. A. Nielsen and I. L. Chuang, *Quantum computation and quantum information: 10th anniversary edition* (Cambridge University Press, 2010).

[100] I. Bengtsson and K. Zyczkowski, *Geometry of quantum states: an introduction to quantum entanglement* (Cambridge University Press, 2006).

[101] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, "Quantum entanglement", Rev. Mod. Phys. **81**, 865–942 (2009).

[102] A. Rényi, "On the dimension and entropy of probability distributions", Acta Mathematica Academiae Scientiarum Hungarica **10**, 193–215 (1959).

[103] A. Rényi, "On measures of entropy and information", in Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, Volume 1: Contributions to the Theory of Statistics (1961), pp. 547–561.

[104] C. Tsallis, "Possible generalization of boltzmann-gibbs statistics", Journal of Statistical Physics **52**, 479–487 (1988).

[105] C. Tsallis, "Nonadditive entropy and nonextensive statistical mechanics - an overview after 20 years", Brazilian Journal of Physics - BRAZ J PHYS **39**, 337–356 (2009).

[106] M. Müller-Lennert, F. Dupuis, O. Szehr, S. Fehr, and M. Tomamichel, "On quantum Rényi entropies: a new generalization and some properties", Journal of Mathematical Physics **54**, 122203 (2013).

[107] V. V. Dodonov, "Nonclassical states in quantum optics: a squeezed review of the first 75 years", Journal of Optics B: Quantum and Semiclassical Optics **4**, R1–R33 (2002).

[108] S. R. Pérez, Y. Hassouni, and P. F. Gonzalez-Diaz, "Coherent states in quantum cosmology", **(arXiv 0709.3302)** (2007).

[109] S. Robles Pérez, Y. Hassouni, and P. González-Díaz, "Coherent states in the quantum multiverse", Physics Letters B **683**, 1–6 (2010).

[110] S. R. Pérez and P. F. Gonzalez-Diaz, "Quantum entanglement in the multiverse", **(arXiv 1111.4128)** (2012).

[111] S. Robles Pérez, A. Balcerzak, M. P. Dąbrowski, and M. Krämer, "Interuniversal entanglement in a cyclic multiverse", Phys. Rev. D **95**, 083505 (2017).

[112] D. F. Walls and G. J. Milburn, *Quantum optics*, 2nd (Springer, Berlin, 2008).

[113] D. Han, Y. S. Kim, and M. E. Noz, “Illustrative example of Feynman’s rest of the universe”, American Journal of Physics **67**, 61–66 (1999).

[114] S. J. R. Pérez, “Restoration of matter-antimatter symmetry in the multiverse”, **(arXiv 1706.06304)** (2017).

[115] L. V. Ahlfors, *Complex analysis, An introduction to the theory of analytic functions of one complex variable*, 2nd ed. (McGraw-Hill Book Company, 1966).

[116] E. Joos, H. Zeh, C. Kiefer, D. Giulini, J. Kupsch, and I. Stamatescu, *Decoherence and the appearance of a classical world in quantum theory*, 2nd ed. (Springer, 1996).

[117] M. Bouhmadi López, C. Kiefer, and M. Krämer, “Resolution of type IV singularities in quantum cosmology”, Phys. Rev. D **89**, 064016 (2014).

[118] F. G. Alvarenga, J. C. Fabris, N. A. Lemos, and G. A. Monerat, “Quantum cosmological perfect fluid models”, General Relativity and Gravitation **34**, 651–663 (2002).

[119] T. Christodoulakis and N. Dimakis, “Classical and quantum Bianchi type III vacuum Hořava–Lifshitz cosmology”, Journal of Geometry and Physics **62**, 2401–2413 (2012).

[120] B. Vakili, “Classical and quantum dynamics of a perfect fluid scalar-metric cosmology”, Physics Letters B **688**, 129–136 (2010).

[121] S. Barroso-Bellido, “Effects of a quantum or classical scalar field on the entanglement entropy of a pair of universes”, Phys. Rev. D **104**, 106009 (2021).

[122] Y. B. Zeldovich, “A Hypothesis, unifying the structure and the entropy of the universe”, Monthly Notices of the Royal Astronomical Society **160**, 1P–3P (1972).

[123] P.-H. Chavanis, “Cosmology with a stiff matter era”, Phys. Rev. D **92**, 103004 (2015).

[124] M. P. Dąbrowski, “Oscillating Friedman cosmology”, Annals of Physics **248**, 199–219 (1996).

[125] P. W. Graham, B. Horn, S. Kachru, S. Rajendran, and G. Torroba, “A simple harmonic universe”, Journal of High Energy Physics **2014**, 29 (2014).

[126] A. T. Mithani and A. Vilenkin, "Collapse of simple harmonic universe", Journal of Cosmology and Astroparticle Physics **2012**, 028–028 (2012).

[127] P. W. Graham, B. Horn, S. Rajendran, and G. Torroba, "Exploring eternal stability with the simple harmonic universe", Journal of High Energy Physics **2014**, 163 (2014).

[128] A. T. Mithani and A. Vilenkin, "Instability of an emergent universe", Journal of Cosmology and Astroparticle Physics **2014**, 006–006 (2014).

[129] A. T. Mithani and A. Vilenkin, "Stabilizing oscillating universes against quantum decay", Journal of Cosmology and Astroparticle Physics **2015**, 010–010 (2015).

[130] M. P. Dąbrowski and A. L. Larsen, "Quantum tunneling effect in oscillating Friedmann cosmology", Phys. Rev. D **52**, 3424–3431 (1995).

[131] T. Damour and A. Vilenkin, "Quantum instability of an oscillating universe", Phys. Rev. D **100**, 083525 (2019).

[132] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, July 2000).

[133] M. P. Dąbrowski and T. Denkiewicz, "Exotic-singularity-driven dark energy", AIP Conference Proceedings **1241**, 561–570 (2010).

[134] M. P. Dąbrowski, "Are singularities the limits of cosmology?", **(arXiv 1407.4851)** (2021).

[135] M. P. Dąbrowski and K. Marosek, "Non-exotic conformal structure of weak exotic singularities", General Relativity and Gravitation **50** (2018).

[136] M. P. Dąbrowski, K. Marosek, and A. Balcerzak, "Standard and exotic singularities regularized by varying constants", **(arXiv 1308.5462)** (2013).

[137] A. Balcerzak, S. Barroso-Bellido, M. P. Dąbrowski, and S. Robles Pérez, "Entanglement entropy at critical points of classical evolution in oscillatory and exotic singularity multiverse models", Phys. Rev. D **103**, 043507 (2021).

[138] F. J. Tipler, "Singularities in conformally flat spacetimes", Physics Letters A **64**, 8–10 (1977).

[139] A. Krolak, “Towards the proof of the cosmic censorship hypothesis”, Classical and Quantum Gravity **3**, 267–280 (1986).

[140] R. Caldwell, “A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of state”, Physics Letters B **545**, 23–29 (2002).

[141] M. P. Dąbrowski, T. Stachowiak, and M. Szydłowski, “Phantom cosmologies”, Phys. Rev. D **68**, 103519 (2003).

[142] S. Nojiri, S. D. Odintsov, and S. Tsujikawa, “Properties of singularities in the (phantom) dark energy universe”, Phys. Rev. D **71**, 063004 (2005).

[143] J. D. Barrow, “Sudden future singularities”, Classical and Quantum Gravity **21**, L79–L82 (2004).

[144] A. Y. Kamenshchik, C. Kiefer, and B. Sandhöfer, “Quantum cosmology with a big-brake singularity”, Phys. Rev. D **76**, 064032 (2007).

[145] S. Nojiri, S. D. Odintsov, and S. Tsujikawa, “Properties of singularities in the (phantom) dark energy universe”, Phys. Rev. D **71**, 063004 (2005).

[146] P. H. Frampton, K. J. Ludwick, and R. J. Scherrer, “The little rip”, Phys. Rev. D **84**, 063003 (2011).

[147] M. C. Bento, O. Bertolami, and A. A. Sen, “Generalized Chaplygin gas, accelerated expansion, and dark-energy-matter unification”, Physical Review D **66** (2002).

[148] T. M. Davis, P. C. W. Davies, and C. H. Lineweaver, “Black hole versus cosmological horizon entropy”, Classical and Quantum Gravity **20**, 2753–2764 (2003).

[149] M. M. Wilde, *Quantum information theory*, 2nd ed. (Cambridge University Press, 2017).

[150] M. A. Castagnino, A. Gangui, F. D. Mazzitelli, and I. I. Tkachev, “Third quantization, decoherence and the interpretation of quantum gravity in minisuperspace”, Class. Quant. Grav. **10**, 2495–2504 (1993).

[151] M. P. Dąbrowski, “Anthropic selection of physical constants, quantum entanglement, and the multiverse falsifiability”, Universe **5**, 172 (2019).

[152] S. Barroso-Bellido and M. P. Dabrowski, “Observational imprints of our lost twin anti-universe”, **(arXiv 2203.07069)** (2022).

[153] Planck Collaboration, “Planck 2018 results - I. Overview and the cosmological legacy of Planck”, A&A **641**, A1 (2020).

[154] S. Robles Pérez, A. Alonso Serrano, C. Bastos, and O. Bertolami, “Vacuum decay in an interacting multiverse”, Physics Letters B **759**, 328–335 (2016).

[155] S. Coleman and F. De Luccia, “Gravitational effects on and of vacuum decay”, Phys. Rev. D **21**, 3305–3315 (1980).

[156] B. A. Bassett, S. Tsujikawa, and D. Wands, “Inflation dynamics and reheating”, Rev. Mod. Phys. **78**, 537–589 (2006).

[157] J. Morais, M. Bouhmadi López, M. Krämer, and S. Robles Pérez, “Pre-inflation from the multiverse: can it solve the quadrupole problem in the cosmic microwave background?”, The European Physical Journal C **78**, 240 (2018).

[158] V. Mukhanov, “Quantum theory of cosmological perturbations in R2 gravity”, Physics Letters B **218**, 17–20 (1989).

[159] D. J. Fixsen, “The temperature of the cosmic microwave background”, The Astrophysical Journal **707**, 916–920 (2009).

[160] M. P. Dąbrowski and T. Stachowiak, “Phantom Friedmann cosmologies and higher-order characteristics of expansion”, Annals of Physics **321**, 771–812 (2006).

[161] M. Dunajski and G. Gibbons, “Cosmic jerk, snap and beyond”, Classical and Quantum Gravity **25**, 235012 (2008).

[162] S. B. Bellido and F. Wagner, “New guest in the third quantized multiverse”, Phys. Rev. D **105**, 106001 (2022).

[163] S. Capozziello and M. De Laurentis, “Extended theories of gravity”, Physics Reports **509**, 167–321 (2011).

[164] S. Capozziello and F. Bajardi, “Nonlocal gravity cosmology: an overview”, International Journal of Modern Physics D (2021).

Streszczenie rozprawy doktorskiej

Uniwersytet Szczeciński – Institut Fizyki – mgr. Samuel Barroso Bellido

Tytuł: What Could a Pair of Universes Tell Us About the Multiverse?

Promotor: prof. dr. hab. Mariusz Dąbrowski

Celem rozprawy doktorskiej jest falsyfikacja koncepcji multiwszechświata uzyskanej w ramach procedury trzeciego kwantowania w formaliźmie kanonicznej kwantowej grawitacji. Ta procedura w sposób naturalny przewiduje kreację pary wszechświatów a zatem w szczególności możliwe istnienie bliźniaczego antywszechświata do tego, w którym my zamieszkujemy. Poprzez intensywne badanie kwantowego splątania pomiędzy naszym oraz bliźniaczym wszechświatem dla różnych modeli, wyciągamy wniosek, iż splątanie odgrywa istotną rolę w takich punktach ewolucji jak: osobliwość początkowa, maksima i minima ekspansji, a także niektóre egzotyczne osobliwości jak m.in. Małe Rozerwanie. Otrzymane rezultaty pozwalają na wymodelowanie oddziaływania pomiędzy parą wszechświatów i skonstruowanie półklasycznego równania Friedmanna pozwalającego na określenie dynamiki naszego wszechświata z uwzględnieniem kwantowego splątania. To pozwala na obliczenie wpływu istnienia hipotetycznego bliźniaczego wszechświata na spektrum obserwowanego kosmicznego mikrofalowego promieniowania tła. Porównanie z danymi z satelity Planck daje ograniczenie na stałą sprzężenia definującą oddziaływanie pary wszechświatów jako $\lambda_o \lesssim \mathcal{O}(10^{-56})\,\mathrm{s}^{-3}$. Na zakończenie wprowadzamy udoskonalenie procedury trzeciego kwantowania poprzez wprowadzenie nowej cząstki pośredniczącej dla oddziaływania wszechświatów w multiwszechświecie. W ten sposób trzecie kwantowanie nabiera zupełnych cech kwantowej teorii pola, co jest zgodne z zasadniczą intencją tej rozprawy.

Słowa Kluczowe: Entropia Śplątania, Kosmologia Kwantowa, Trzecie Kwantowanie.

Data, podpis: 20/05/2022 Samuel Barroso Bellido

Summary of the Doctoral Thesis

University of Szczecin – Institute of Physics – Samuel Barroso Bellido, M. Sc.
Title: What Could a Pair of Universes Tell Us About the Multiverse?
Supervisor: prof. dr. hab. Mariusz Dąbrowski

The present work is aimed to falsify the multiverse as it is prescribed by the Third Quantization formalism of Canonical Quantum Gravity. The formalism naturally predicts pair creation of universes and so the possible existence of a twin antiuniverse of that we inhabit. We extensively study the quantum entanglement between them for different models, finding that it is relevant at the initial singularity, at the maxima and minima of expansion, and at some exotic singularities like the Little Rip singularity. We use the conclusions found from the entanglement research to constraint the interaction between our universe and its twin and thus recreate the semiclassical Friedmann equation from where we obtain the dynamics of the universe which takes the entanglement effects into account. We then apply it in order to get the observational imprints of our hypothetical twin antiuniverse on the spectrum of the cosmic microwave background. For the case we consider, the constant coupling which governs the strength of the interaction is calculated to be $\lambda_o \lesssim \mathcal{O}(10^{-56})\,\mathrm{s}^{-3}$ in order to reproduce the angular power spectrum obtained by Planck satellite. We finish by completing the Third Quantization formalism including a new particle in the multiverse accountable for the interaction between universes. That way, Third Quantization formalism is the true Quantum Field Theory it was supposed to be by construction.

Keywords: Entanglement Entropy, Quantum Cosmology, Third Quantization.

Date, signature: 20/05/2022 Samuel Barroso Bellido