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Title of the dissertation: **Asymptotics analysis and analytic tools for certain types of C_0-semigroups**

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Dissertation summary

The object of this study was the analysis of asymptotic behavior of a certain class of unbounded C_0-semigroups and, independently, the extension of some existing results concerning the delay differential equations of the neutral type in $\mathbb{C}^n$ to the infinite-dimensional case.

In the first part of the dissertation, we prove that for the C_0-semigroup $\{T(t)\}_{t\geq 0}$ with the generator A having some particular asymptotic properties when truncated to the images of the Riesz projections of the operator A associated with certain subsets of the spectrum,

$$\lim_{t\to\infty}\frac{\|T(t)A^{-1}\|}{f(t)}=0.$$

The real function $f(t)$ in some sense approximates the norm of the semigroup $\|T(t)\|$ and, for regular enough C_0-semigroups, the function $f(t)$ can equal $\|T(t)\|$. This property means that the classical solutions of the corresponding Cauchy problem grow slower (or decay faster) than the norm of the semigroup. Our results extend some existing ones, mainly by allowing the spectrum of the generator to be located on the the axis $\{z\in\mathbb{C}:\operatorname{Re}(z)=\omega_0\}$.

In the second part of the dissertation we consider the differential equation

$$\dot{z}(t)=A\dot{z}(t-1)+\int_{-1}^{0}A_2(\theta)\dot{z}(t+\theta)\mathrm{d}\theta+\int_{-1}^{0}A_3(\theta)z(t+\theta)\mathrm{d}\theta,\ z(t)\in H$$

where H is an arbitrary separable Hilbert space and $A, A_2(\theta), A_3(\theta)$ are bounded linear operators with some particular properties. We extend the results which hold for the finite-dimensional case including the generation of a C_0-semigroup by the linear operator $\mathcal{A}$ representing the above equation and the existence of a Riesz basis of the corresponding space $H\times L^2([-1,0];H)$ constructed from the Riesz projections of the operator $\mathcal{A}$.

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