MAX-PLANCK-INSTITUT
FÜR DYNAMIK KOMPLEXER
TECHNISCHER SYSTEME
MAGDEBURG

CSC
NUMERISCHE METHODEN IN DER
SYSTEM- UND REGELUNGSTHEORIE

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Review of the doctoral dissertation of Bartosz Wasilewski
"Asymptotics analysis and analytic tools for certain types of C_0-semigroups"

This dissertation of Mr. Bartosz Wasilewski develops the spectral theory of linear operators for the analysis of large-time behavior of C_0-semigroups in Banach and Hilbert spaces. Semigroup theory of linear operators is a significant part of modern functional analysis and plays a crucial role in the mathematical formulation of various applied problems. At its core, this theory provides a powerful framework for understanding the evolution of systems over time, particularly those that can be modeled by differential equations. By representing the dynamics of such systems as semigroups, this theory allows for a deeper analysis of their behavior in infinite-dimensional spaces. This is indispensable in fields like quantum mechanics, mechanics of flexible structures, fluid dynamics, process engineering, and chemical engineering, where systems often exhibit complex, time-dependent behavior. Furthermore, semigroup theory offers efficient methods for addressing problems of stability, controllability, and observability in systems and control theory.

The dissertation of Mr. Wasilewski is composed of three chapters: Chapter 1 presents some basic constructions from functional analysis and complex analysis, while the main results are presented in Chapters 2-3.

Chapter 2 addresses the fundamental issue of the asymptotic behavior of a semigroup $\{T(t)\}_{t\geq 0}$ on a Banach space X, without assuming the spectral condition of the form (2.4) and thereby allowing for the case where

$$(\omega_0 + i\mathbb{R}) \cap \sigma(A) \neq \emptyset. \qquad (*)$$

Here, ω_0 is the growth bound of the semigroup $\{T(t)\}_{t\geq 0}$, $A : D(A) \to X$ is the infinitesimal generator of $\{T(t)\}_{t\geq 0}$, and $\sigma(A)$ is the spectrum of A. In Theorem 52, the author proposes sufficient conditions for characterizing the asymptotic property

$$\lim_{t\to+\infty} \frac{\|T(t)R(\mu; A)\|}{f(t)} = 0 \quad \text{for all } \mu \in \mathbb{C} \setminus \sigma(A), \qquad (**)$$

where $R(\mu; A)$ is the resolvent of A and the function $f(t)$ can be taken as $f(t) = \|T(t)\|$. The proof of Theorem 52 exploits the Riesz projection technique and the quotient space defined by an appropriate seminorm. An interesting example (Example 51) illustrates that property $(**)$ is possible under condition $(*)$.

In Chapter 3, a class of delay differential equations of the neutral type in a Hilbert space H is studied within the framework of spectral theory. First, the existence and uniqueness of classical solutions is established by the Banach fixed point theorem, and the semigroup generation property is proved in Theorem 61. Then the main result on the existence of a Riesz basis is summarized in Theorem 74.

The thesis is overall well-composed, featuring clear and well-defined theoretical results. The reviewer's comments are as follows.

1. The presented method of proving the Riesz basis property (e.g., in Lemma 69) with the use of quadratically close sequences could be compared with the approaches described in Sections 2.7-2.10 of the book by B.-Z. Guo and J.-M. Wang "Control of Wave and Beam PDEs", Springer, 2019.

2. It would be reasonable to comment on how large the class of functions $f(t)$ satisfying conditions (2.6) and (2.7) in Theorem 52 is. In particular, are there illustrative examples of non-exponential functions that satisfy these conditions if $\omega_0 \neq 0$?

3. There are typos scattered in the text: "the the axis" (p. 3, second to last paragraph); "this results" (p. 24, second paragraph); "this results" (p. 57, first paragraph); "opearator" (p. 65, end of the proof of Theorem 74). The use of hyphenation in compound terms like "nonzero" (Definition 17) and "non-zero" (proof of Theorem 52) should be consistent.

The comments provided above do not detract from the exceptional quality of Mr. Wasilewski's work.

This thesis offers an original addition to the theory of unbounded semigroups of linear operators, utilizing spectral methods and perturbation theory. Additionally, the author has conducted a comprehensive analysis of a specific class of delay equations. The submitted thesis fulfills the conditions set by Article 187 ust. 1-3 "ustawy z dnia 20 lipca 2018 r. Prawo o szkolnictwie wyższym i nauce (Dz. U. z 2023 r. poz. 742 ze zm.)".

In conclusion, I endorse Mr. Wasilewski's dissertation for the defense process and rate his thesis with the highest possible grade.

Yours respectfully,

Alexander Zuyev

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